Electromechanical analysis of a piezoresistive pressure microsensor for low-pressure biomedical applications

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The electromechanical analysis of a piezoresistive pressure microsensor with a square-shaped diaphragm for low-pressure biomedical applications is presented. This analysis is developed through a novel polynomial model and a finite element method (FEM) model. A microsensor with a diaphragm 1000 \( \mu \text{m} \) length and with three different thicknesses (10, 15, and 20 \( \mu \text{m} \)) is studied. The electric response of this microsensor is obtained with a Wheatstone bridge of four p-type piezoresistors located on the diaphragm surface. The diaphragm that is 10 \( \mu \text{m} \) thick exhibits a maximum deflection of 3.74 \( \mu \text{m} \) using the polynomial model, which has a relative difference of 5.14 and 0.92\% with respect to the Timoshenko model and the FEM model, respectively. The maximum sensitivity and normal stress calculated using the polynomial model are 1.64 mV/V/kPa and 102.1 MPa, respectively. The results of the polynomial model agree well with the Timoshenko model and FEM model for small deflections. In addition, the polynomial model can be easily used to predict the deflection, normal stress, electric response and sensitivity of a piezoresistive pressure microsensor with a square-shaped diaphragm under small deflections.

Keywords: Finite element model; piezoresistors; polynomial model; pressure microsensor.

Descubiertos: Modelo de elemento finito; piezoresistores; modelo polinomial; microsensor de presión.

1. Introduction

Pressure microsensors are widely used in automotive applications, process control and biomedical applications [1]. Pressure microsensors used in biomedical applications include the measurement of blood pressure [2], intraocular eye pressure [3], intracranial pressure, pulse rate, intrauterine pressure, abdominal and urinary pressure [4-5]. For many biomedical applications, the capacitive detection technique is used mainly due to its high sensitivity. However, the capacitive pressure microsensors have problems with the hermetic vacuum scaling of the capacitive cavity, the electrical lead transfer between the vacuum-sealed cavity and the outside world [6], the high cost due to the complex fabrication process and the difficult to use post-end circuits to compensate the low linearity of these microsensors [7]. To overcome these problems, piezoresistive pressure microsensors are an other option for designers and researchers because these microsensors are easy to use and to fabricate [8-9]. In addition, the low sensitivity of piezoresistive pressure microsensors can be improved by integrating amplifier circuits [10].

The pressure microsensors often use a thin square-shaped diaphragm as their main sensor element. This is because of its compatibility with bulk and surface silicon micromachining processes [11-12]. A pressure applied on the diaphragm generates an increase in its deflection until the elastic force is balanced by the pressure. The pressure range that can be measured by the diaphragm depends on its dimensions (surface area and thickness), geometry, edge conditions, and material [13]. For example, in biomedical applications to measure the blood pressure and heart rate, pressure microsensors are required to operate in the range of 0-40 kPa (0-300 mmHg) [14].
The diffused resistors on the silicon substrate are used to measure the strain of the diaphragm of the pressure microsensors. This piezoresistive microsensor generally has four piezoresistors in a Wheatstone bridge configuration to measure the stresses in a silicon diaphragm under normal pressure [15].

The electromechanical behavior of piezoresistive pressure microsensors is predicted during the design phase. This design is used to find the maximum electromechanical performance of the microsensors to improve their sensitivity and resolution. In the past, the electromechanical design of these devices has often been studied with the Timoshenko model for plates [16-17] and finite element method (FEM) models [18-19]. However, the Timoshenko model contains complicated terms and FEM models need considerable computing time. Furthermore, the accuracy of a FEM model depends on the shape and size of mesh used in these models; thus, FEM models are difficult to use between designers and researchers. Therefore, simple theoretical models are needed to decrease the design time of pressure microsensors for biomedical applications. In order to solve this problem, this paper presents a novel polynomial model for an easier and faster prediction of the electromechanical behavior of a piezoresistive pressure microsensor with a thin square-shaped diaphragm, which is proposed to measure the blood pressure and heart rate.

The paper is organized as follows. In Sec. 2, a novel polynomial model for predicting the electromechanical behavior of a piezoresistive pressure microsensor with a square-shaped diaphragm is developed. This model is obtained with the small-deflection theory for the bending of plates and the Ritz method. In addition, the electromechanical response calculated with the polynomial model is compared with the Timoshenko model for plates. In Sec. 3, the discussion of the electromechanical behavior of a piezoresistive pressure microsensor obtained with the polynomial model, Timoshenko model, and FEM model is presented.

2. Pressure microsensor design

The mechanical and electric design of a piezoresistive pressure microsensor with a square-shaped diaphragm to measure blood pressure and heart rate is needed to improve its electromechanical performance. Therefore, this design will help in choosing the dimensions of the microsensor with the best sensitivity and resolution for these biomedical applications.

2.1. Mechanical design

The diaphragm of a piezoresistive pressure microsensor can be modeled as a square plate with four edges clamped under a uniform normal pressure. In this work, the thin diaphragm of the piezoresistive pressure microsensor is considered as a thin plate with edges clamped. A plate is called “thin” when its ratio of thickness to the smaller span length is less than 1/20 [20]. Figure 1 shows the complete and cross-sectional views of a typical piezoresistive pressure microsensor with a thin square-shaped diaphragm. This microsensor has a Wheatstone bridge with four p-type piezoresistors, which are located near the edges of the diaphragm. The diaphragm and piezoresistors are aligned with the (110) directions in the (100) crystallographic plane.

The governing equation of the deflection and normal stress of a thin diaphragm is derived considering the fundamental assumptions (also known as Kirchhoff assumptions) of the small-deflection theory for the bending of thin plates, which are stated as follows [20]:

1. The material of the plate is elastic, homogeneous, and isotropic.
2. The plate is initially flat.
3. The deflection of the midsurface is small compared with the thickness of the plate and a maximum deflection of one-fifth of its thickness is considered the limit of the small-deflection theory. The slope of the deflected surface is very small and the square of the slope is a negligible quantity with respect to unity.

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2. Pressure microsensor design

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2.1. Mechanical design
and shear stresses \( (\tau_{xy}, \tau_{xz} \text{ and } \tau_{yz}) \) \cite{21}. That is,

\[
\begin{align*}
\begin{bmatrix}
M_x \\
M_y \\
M_{xy} 
\end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy} 
\end{bmatrix} dz,
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
Q_x \\
Q_y 
\end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix}
\tau_{xz} \\
\tau_{yz} 
\end{bmatrix} dz,
\end{align*}
\]

\noindent where \( z \) is the vertical distance measured from the diaphragm midsurface.

Based on the reciprocity law of shear stresses \( (\tau_{xy} \text{ and } \tau_{yz}) \), the twisting moments on perpendicular faces of a diaphragm element are identical, i.e., \( M_{xy} = M_{yx} \). It is important to mention that while the theory of thin plates omits the effect of the strain components \( \gamma_{xz} \text{ and } \gamma_{yz} \) on bending, the vertical shear forces \( Q_x \text{ and } Q_y \) are not negligible.

Figure 2b shows the equilibrium of an element cut out of a diaphragm, under a distributed load, by two pairs of planes parallel to the \( xz \) and \( yz \) planes, since the stress-resultants and stress-couples are considered at the midsurface of this element. Note that as this element is very small, the force and moment components are distributed uniformly on the midsurface of the diaphragm element. Projecting all the forces on the element in the \( z \)-axis, the following equations of equilibrium are obtained \cite{21}:

\[
\begin{align*}
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q &= 0 \\
\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y &= 0 \\
\frac{\partial M_{xy}}{\partial y} + \frac{\partial M_x}{\partial x} - Q_x &= 0,
\end{align*}
\]

where \( q(x,y) \) is a uniform load applied at the diaphragm surface. From equation set (2), the relation between the uniform load and the bending moment can be rewritten as \cite{21}

\[
\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q. 
\]

Substituting equation set (1) into (3), a relation between normal stress and uniform load can be obtained as

\[
\int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial x \partial y} + \frac{\partial^2 \sigma_y}{\partial y^2} \right) dz = -q. 
\]
the following boundary conditions of a clamped diaphragm:

\[
\begin{align*}
    w &= 0 \quad (x = x = a) \\
    w &= 0 \quad (y = y = b). 
\end{align*}
\]

The solution of the partial differential Eq. (6) is complicated and this solution is needed to find the deflection and the normal stress components of the diaphragm. Double trigonometric-series solutions can be used to solve Eq. (6), but generally are not easy to use [22]. However, the polynomial solutions are the simplest equations to solve equations similar to Eq. (6); however, they must be obtained carefully to satisfy the boundary conditions indicated by Eq. (8) and to keep acceptable accuracy. Therefore, in this work a novel polynomial model is proposed for solving Eq. (6) with the boundary conditions of a clamped diaphragm. Thus, the proposed polynomial model is given by

\[
w(x, y) = \sum_{m=1}^{r} \sum_{n=1}^{s} c_{mn} \left[ 1 - \frac{x}{a} \right]^{2} \times \left[ 1 - \frac{y}{b} \right]^{2} \left( \frac{x}{a} \right)^{2m} \left( \frac{y}{b} \right)^{2n},
\]

where \( w \) is the deflection, \( c_{mn} \) represents coefficients to be determined, \( a \) and \( b \) are the lengths of the diaphragm edges.

The novel polynomial model satisfies the boundary conditions indicated by Eq. (8) and contains two series of polynomial terms. These polynomial terms have unknown coefficients \( c_{mn} \) that can be found by variational methods. The polynomial model considers the deflection of a diaphragm as the superposition of polynomial curves of order 2\( m \) and 2\( n \) in the \( x \)- and \( y \)-directions. Furthermore, the \( r \) and \( s \) terms indicate the maximum number of polynomial curves in the \( x \)- and \( y \)-directions to use in the polynomial model. The maximum value of these terms (\( r \) and \( s \)) will depend on the designer and the variational method used to find the coefficients \( c_{mn} \).

The Ritz method is a variational method based on the principle of minimum potential energy to solve boundary value problems of plates. In this work, the Ritz method is considered to find the coefficients \( c_{mn} \) of the polynomial model. First, this model is applied to a pressure microsensor with a rectangular-shaped diaphragm and afterwards is simplified to a square-shaped diaphragm. Therefore, the Ritz method is applied to a rectangular-shaped diaphragm with sides \( a \) and \( b \) under a uniform surface load. Thus, the strain energy \( U \) associated with the bending of the diaphragm is given by [22]

\[
U = \frac{1}{2} \int_{A} D \left\{ \left( \frac{\partial^{2} w}{\partial x^{2}} + \frac{\partial^{2} w}{\partial y^{2}} \right)^{2} - 2(1 - \nu) \times \left[ \frac{\partial^{2} w}{\partial x^{2}} \frac{\partial^{2} w}{\partial y^{2}} - \left( \frac{\partial^{2} w}{\partial x \partial y} \right)^{2} \right] \right\} dxdy,
\]

where \( A \) is the area of the diaphragm surface.

The work done by the surface load \( q(x, y) \) is

\[
W = \int_{A} wqdxdy.
\]
The potential energy equation is obtained by \( \Pi = U - W \):

\[
\Pi = \int \int_{A} \left\{ \frac{D}{2} \left[ \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1 - v) \right] \right. \\
\times \left. \left[ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] - wq \right\} \, dx \, dy \ . \tag{12}
\]

Assuming the boundary conditions given by Eq. (8) and integrating the last term of Eq. (10) by parts, then the strain energy \( U \) is reduced to

\[
U = \frac{D}{2} \int \int_{A} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 \, dx \, dy . \tag{13}
\]

Introducing Eq. (9) into Eq. (13) yields

\[
U = \int_{0}^{b} \int_{a}^{x} \left\{ \sum_{m=1}^{r} \sum_{n=1}^{s} c_{mn} \left( x \right)^{2m} \left( y \right)^{2n} \times \left[ \left( 1 - \frac{y}{b} \right)^2 \left( \frac{2m(2m-1)}{b^2} \right) \left( 1 - \frac{x}{a} \right)^{-1} \left( \frac{x}{a} \right)^{-1} + \frac{2}{a^2} \right] \right. \\
\left. - \frac{8n}{a^2} \left[ 1 - \frac{x}{a} \right] \left( \frac{x}{a} \right)^{-1} + \frac{2}{a^2} \right] \\
\left. + \left[ 1 - \frac{x}{a} \right]^2 \left( \frac{2n(2n-1)}{b^2} \right) \left( 1 - \frac{y}{b} \right)^{-1} \left( \frac{y}{b} \right)^{-2} \right) \times \left[ \left( 1 - \frac{y}{b} \right)^2 \left( \frac{4y}{b} \right) \left( 1 - \frac{y}{b} \right)^{-1} + \frac{2}{b^2} \right] \right\} \, dx \, dy \tag{14}
\]

The work done by the uniform surface load \( q(x,y) = q_0 \) on a rectangular-shaped diaphragm is calculated by

\[
W = q_0 \int_{0}^{b} \int_{a}^{x} \sum_{m=1}^{r} \sum_{n=1}^{s} c_{mn} \left[ 1 - \frac{x}{a} \right]^2 \times \left[ \left( 1 - \frac{y}{b} \right)^2 \left( \frac{2m}{b^2} \right) \left( \frac{x}{a} \right)^{-1} \left( \frac{y}{b} \right)^{-2} \right] \, dx \, dy \ . \tag{15}
\]

Then, Eqs. (14) and (15) are substituted into Eq. (12) and the unknown coefficients \( c_{mn} \) are determined by the minimum potential energy principle. Thus,

\[
\frac{\partial \Pi}{\partial c_{mn}} = 0 . \tag{16}
\]

We use the first value of \( r \) and \( s \) of the polynomial model to obtain the simplest expression of \( c_{mn} \); consequently, the coefficient \( c_{11} \) is obtained as

\[
c_{11} = \frac{49a^4b^4q_0}{8D(7b^4 + 4a^2b^2 + 7a^4)} . \tag{17}
\]

Then, the deflection obtained with the polynomial model is given by

\[
w = \frac{49a^4b^4q_0}{8D(7b^4 + 4a^2b^2 + 7a^4)} \times \left[ 1 - \frac{x}{a} \right]^2 \left[ 1 - \frac{y}{b} \right]^2 \left( \frac{x}{a} \right)^2 \left( \frac{y}{b} \right)^2 . \tag{18}
\]

For the case of a square-shaped diaphragm \( (a = b) \), the maximum deflection \( (w_{max}) \) is found at its center:

\[
w_{max} = \frac{49q_0a^4}{36864D} . \tag{19}
\]

The normal stress components \( (\sigma_x \text{ and } \sigma_y) \) of a rectangular-shaped diaphragm are found through the substitution of Eq. (18) into Eq. (5):

\[
\sigma_x = \frac{147a^2b^2q_0}{h^3(7b^4 + 4a^2b^2 + 7a^4)} \times \left[ \left( \frac{x}{a} \right)^2 - \frac{4x}{a} \left( \frac{1 - \frac{x}{a}}{\frac{1}{a}} \right) + a^2 \left( \frac{1 - \frac{x}{a}}{\frac{1}{a}} \right)^2 \right] \times \left[ \left( \frac{y}{b} \right)^2 - \frac{4y}{b} \left( \frac{1 - \frac{y}{b}}{\frac{1}{b}} \right) + \frac{(1 - \frac{y}{b})^2}{\frac{1}{b}} \right] . \tag{20}
\]

\[
\sigma_y = \frac{147a^2b^2q_0}{h^3(7b^4 + 4a^2b^2 + 7a^4)} \times \left[ \left( \frac{y}{b} \right)^2 - \frac{4y}{b} \left( \frac{1 - \frac{y}{b}}{\frac{1}{b}} \right) + \frac{(1 - \frac{y}{b})^2}{\frac{1}{b}} \right] \times \left[ \left( \frac{x}{a} \right)^2 - \frac{4x}{a} \left( \frac{1 - \frac{x}{a}}{\frac{1}{a}} \right) + a^2 \left( \frac{1 - \frac{x}{a}}{\frac{1}{a}} \right)^2 \right] . \tag{21}
\]

The maximum normal stresses are found at the middle edges of the rectangular-shaped diaphragm at its upper surface \( (z = h/2) \) and are given by

\[
\sigma_{x_{max}} = \frac{147a^2b^4q_0}{32h^2(7b^4 + 4a^2b^2 + 7a^4)} \]

\[
\sigma_{y_{max}} = \frac{147a^4b^2q_0}{32h^2(7b^4 + 4a^2b^2 + 7a^4)} . \tag{22}
\]

The maximum normal stresses for a square-shaped diaphragm are found with the substitution \( a = b \) into Eq. (22).

The Timoshenko model [23] for a rectangular-shaped plate under small deflections is used to compare its results in relation to the polynomial model. Based on the small-deflection theory for the bending of plates, Timoshenko [23] assumed the total deflection of a rectangular-shaped plate with clamped edges as the sum of three components: \( w_1, w_2, \) and \( w_3 \). The first component, \( w_1 \), comes from the deflection of a simply supported plate under the same pressure load. The following equations are derived for a rectangular-shaped plate \( (a \text{ and } b \text{ width}) \) under a pressure \( q_0 \). In addition, the coordinate system used in these equations is located in the

\[
\]
center of the rectangular-shaped plate. Therefore,

\[
w_1 = \frac{4q_0a^4}{\pi^4D} \sum_{m=1,3,5,\ldots}^{\infty} \frac{(-1)^{(m-1)/2}}{m^6} \cos \left(\frac{m\pi x}{a}\right) \times \left[1 - \frac{\alpha_m \tanh \alpha_m + 2}{2 \cosh \alpha_m} \cosh \left(\frac{m\pi y}{a}\right)\right] + \frac{1}{2 \cosh \alpha_m} \frac{\left(m\pi y\right)}{a} \sinh \left(\frac{m\pi y}{a}\right) (23)
\]

\[
w_2 = \frac{-a^2}{2\pi^2D} \sum_{m=1,3,5,\ldots}^{\infty} \frac{E_m(-1)^{(m-1)/2}}{m^2 \cosh \beta_m} \times \cos \left(\frac{m\pi x}{b}\right) \frac{m\pi y}{b} \sinh \left(\frac{m\pi x}{b}\right) - \beta_m \tanh \beta_m \cos \left(\frac{m\pi y}{b}\right) (24)
\]

where \(E_m\) and \(F_m\) are coefficients to be determined. Also,

\[
\alpha_m = \frac{m\pi b}{2a} \quad \text{and} \quad \beta_m = \frac{m\pi a}{2b}. (26)
\]

Hence, the total deflection of a rectangular-shaped diaphragm (plate) under small deflections is calculated by

\[
w_t = w_1 + w_2 + w_3. (27)
\]

The values of \(E_m\) and \(F_m\) can be obtained by the following equations:

\[
\frac{4q_0a^2\alpha_n}{\pi^3n^3 \cosh^2 \alpha_n} - \tanh \alpha_n = \frac{E_n}{n} \left(\tanh \alpha_n + \frac{\alpha_n}{\cosh^2 \alpha_n}\right) + \frac{8na}{\pi b} \sum_{m=1,3,5,\ldots}^{\infty} \frac{F_m}{m^3 \left(\frac{a^2}{b^2} + \frac{n^2}{m^2}\right)} (28)
\]

\[
\frac{4q_0b^2\beta_n}{\pi^3n^3 \cosh^2 \beta_n} - \tanh \beta_n = \frac{F_n}{n} \left(\tanh \beta_n + \frac{\beta_n}{\cosh^2 \beta_n}\right) + \frac{8nb}{\pi a} \sum_{m=1,3,5,\ldots}^{\infty} \frac{E_m}{m^3 \left(\frac{a^2}{b^2} + \frac{n^2}{m^2}\right)} (29)
\]

For the case of a square-shaped diaphragm, the following assumptions are considered: \(E_n = F_n\) and the Eqs. (28) and (29) are same. Therefore, Eqs. (28) and (29) are reduced to

\[
\frac{E_n}{n} \left(\tanh \alpha_n + \frac{\alpha_n}{\cosh^2 \alpha_n}\right) + \frac{8n}{\pi} \sum_{m=1,3,5,\ldots}^{\infty} \frac{E_m}{m^3} \times \frac{1}{\left(1 + \frac{n^2}{m^2}\right)^2} = \frac{4q_0a^2n^2}{\pi^3n^3} \left(\frac{\alpha_n}{\cosh^2 \alpha_n} - \tanh \alpha_n\right). (30)
\]

The coefficients \(E_m\) can be determined by the method of successive approximations. Only the first four coefficients \((E_1, E_3, E_5, \text{and } E_7)\) were considered because a greater increase in these terms does not significantly increase the accuracy of the Timoshenko model [23]; thus, \(E_1 = 0.3722\), \(E_3 = -0.0380\), \(E_5 = -0.0178\), and \(E_7 = -0.0085\), where

\[
K = \frac{-4q_0a^2}{\pi^3}. (31)
\]

The normal stress components of a square-shaped plate can be determined by Eq. (5), considering these four coefficients \((E_1, E_3, E_5 \text{ and } E_7)\).

In the next section, the electric design of the piezoresistive pressure microsensor is presented.

### 2.2. Electric design

The piezoresistive pressure microsensor has a Wheatstone bridge with four p-type piezoresistors on the diaphragm surface, as shown in Fig. 3. Two pairs of piezoresistors are placed on opposite sides of the edges of the diaphragm to increase its sensitivity to an applied pressure. Accordingly, two piezoresistors are in parallel with the maximum normal stress \((\sigma_x)\) and the other two are perpendicular to \(\sigma_x\). For a piezoresistor subjected to parallel and perpendicular stress components \((\sigma_1 \text{ and } \sigma_t)\), the change in resistance is [24]

\[
\frac{\Delta R}{R} = \pi_t \sigma_t + \pi_t \sigma_t, (32)
\]

where \(\pi_t\) and \(\pi_t\) are the piezoresistive coefficients parallel and perpendicular to the piezoresistor length.

The values of the piezoresistive coefficients depend on the orientation of the wafer and the diaphragm, the type and concentration of doping, and temperature [25]. For the (110) directions in the (100) crystallographic plane, the parallel and perpendicular piezoresistive coefficients are given by [24]

\[
\pi_t = \pi_{11} - 2(\pi_{11} - \pi_{12} - \pi_{44}) \left(\frac{1}{4}\right) \quad \pi_t = \pi_{12} + (\pi_{11} - \pi_{12} - \pi_{44}) \left(\frac{1}{2}\right) (33)
\]

where \(\pi_{11}, \pi_{12} \text{ and } \pi_{44}\) are the fundamental cubic piezoresistive coefficients. This work considered a resistivity of 7.8 \Omega \cm for the wafer, p-type piezoresistors with \(\pi_{11} = 6.6 \times 10^{-11}\) \Pa\(^{-1}\), \(\pi_{12} = -1.1 \times 10^{-11}\) \Pa\(^{-1}\) and \(\pi_{44} = 138.1 \times 10^{-11}\) \Pa\(^{-1}\) [15]. Thus, the parallel and perpendicular piezoresistive coefficients are \(\pi_t = 71.8 \times 10^{-11}\) \Pa\(^{-1}\) and \(\pi_t = -66.3 \times 10^{-11}\) \Pa\(^{-1}\). Besides, \(E = 169.8\) GPa was considered for the silicon and Poisson’s ratio \(\nu = 0.066\) [24].

The output voltage, \(\Delta V\), of the Wheatstone bridge with a supply voltage, \(V_{in}\), is given by [24]

\[
\frac{\Delta V}{V_{in}} = \frac{(\Delta R/R)_t - (\Delta R/R)_t}{2 + (\Delta R/R)_t + (\Delta R/R)_t} (34)
\]

where

\[
\left(\frac{\Delta R}{R}_t\right) = \pi_t \sigma_t + \pi_t \sigma_t \left(\frac{\Delta R}{R}_t\right) = \pi_t \sigma_t + \pi_t \sigma_t. (35)
\]
The parallel and perpendicular stress components are calculated using

\[ \sigma_l = \sigma_x(0,b/2) \quad \sigma_t = \sigma_x(a/2,0). \]  

The sensitivity, \( S \), of the pressure microsensor can be determined by the following equation:

\[ S = \frac{\Delta V/V_{in}}{q_0}, \]

where \( q_0 \) is the uniform-normal pressure on the diaphragm surface.

3. Results and discussion

This section shows the result of the relations between the size, pressure, deflection, normal stress components and electric response of a piezoresistive pressure microsensor proposed to measure blood pressure and heart rate. This microsensor has a square-shaped silicon diaphragm 1000 \( \mu \)m length and in three different thicknesses (10, 15, and 20 \( \mu \)m) under a uniform normal pressure. The pressure range considered for the electromechanical analysis of this microsensor was 0-40 kPa (0-300 mmHg) [14]. The electromechanical response was obtained with the novel polynomial model, which agrees well with the Timoshenko model and FEM model for small deflections. In addition, this polynomial model predicts the electromechanical behavior of the piezoresistive pressure microsensor more easily and quickly.

Figure 4 shows the absolute amplitude of the deflection distribution of a square-shaped diaphragm (1000 \( \mu \)m length and 10 \( \mu \)m thickness) of the pressure microsensor, which is obtained with the proposed polynomial model. This deflection distribution is caused by a uniform normal pressure (15 kPa) on the diaphragm’s external surface. The maximum deflection (1.40 \( \mu \)m) is located at the center of this diaphragm. In addition, the absolute deflection distribution over the middle (\( y = b/2 \)) of one quadrant (due to symmetry) of this diaphragm is found using the polynomial model and Timoshenko model, as shown in Fig. 5. For this case three thicknesses (10, 15, and 20 \( \mu \)m) are considered, and a uniform normal pressure of 15 kPa on the diaphragm. The maximum deflection (1.40 \( \mu \)m) is less than one-fifth (2 \( \mu \)m) of the smallest thickness (10 \( \mu \)m), which satisfies the condition for small deflections [23]. The deflections obtained by the model polynomial agree well with the results of the Timoshenko model.

Figure 6 indicates the normal stress distribution (\( \sigma_x \)) of the same square-shaped diaphragm 10 \( \mu \)m thick as a function of \( x \) and \( y \) distances, respectively. This stress distribution was calculated using the polynomial model Eq. (20) and considering a pressure of 15 kPa on the diaphragm. The maximum stresses (38.28 MPa) are found at the middle edges (\( x = 0, y = 500 \mu \text{m} \) and \( x = 1000 \mu \text{m}, y = 500 \mu \text{m} \)) of the
Figure 7. Deflection distribution ($\mu$m) of a FEM model of a piezoresistive pressure microsensor with a square-shaped diaphragm (1000 $\mu$m length) under an uniform normal pressure (15 kPa) and considering thicknesses of 10, 15, and 20 $\mu$m.

The center of the diaphragm is subjected to compressive-type stresses, of which the highest compressive stress was 20.40 MPa.

A FEM model of the same piezoresistive pressure microsensor was made through ANSYS and its mechanical behavior was compared with the polynomial model and the Timoshenko model. The FEM model represents one quarter of the pressure microsensor (due to symmetry conditions) in order to decrease the computer time. First, the model was drawn using CAD software (Solid Edge17) and after that was transferred to ANSYS software. Then, the load and mesh conditions were applied to this FEM model with elements...
type solid95 with three degrees of freedom each. Finally, the FEM model was solved and its electromechanical behavior was obtained for a pressure range from 0 to 40 kPa. This FEM model was made and solved in an approximate time of four hours. This computation time is greater than the time used by the polynomial model and the Timoshenko model. For the polynomial model, it took about 20 minutes to capture Eqs. (9), (20), (34), and (37) with Matlab software and to define the load and material conditions. In addition, the Matlab software needed approximately four seconds to solve these equations. However, the time taken for the Timoshenko model to be captured with Matlab software was about twice as long as the time needed for the polynomial model.

First, the deflection distribution of the FEM model of the microsensor was found by considering a square-shaped diaphragm (1000 µm length) in three different thicknesses (10, 15, and 20 µm). Figure 7 shows the results of the deflection distribution of this microsensor under a uniform normal pressure of 15 kPa. The maximum deflection (1.39 µm) was found in the diaphragm 10 µm thick. This value is 3.29 and 7.60 times greater than that obtained in the diaphragms with 15 and 20 µm of thickness, respectively. In addition, the normal stress distribution for the same diaphragm with three different thicknesses (10, 15, and 20 µm) was recorded, as shown in Fig. 8. The maximum tensile stress (44.01 MPa) is located at the middle edge of the diaphragm 10 µm thick. This value is 2.37 and 4.40 higher than that found in the diaphragms 15 and 20 µm thick, respectively. The stress distribution decreases and becomes compressive at the center of the diaphragm. In both cases, the stresses are less than the rupture stress of 360 MPa in ⟨100⟩ silicon [26].

Figure 9 shows the deflections of the pressure microsensor considering three different thicknesses (10, 15, and 20 µm) and a uniform normal pressure from 0 to 40 kPa. These deflections were obtained at the diaphragm center using the polynomial model, the Timoshenko model and the FEM model. The results of the polynomial model agree well with the other two models. The diaphragm 10 µm thick exhibits a maximum deflection of 3.74 µm using the polynomial model, which has a relative difference of 5.14 and 0.92% with respect to the Timoshenko model and the FEM model, respectively. The deflections for the diaphragms 15 and 20 µm thick showed a reduction of 70.38 and 87.50% with respect to the thickness of 10 µm.

Fig. 9. Maximum deflections variation versus pressure applied to a piezoresistive pressure microsensor with a square-shaped diaphragm (1000 µm length) and considering thicknesses of 10, 15, and 20 µm.

Fig. 10. Maximum normal stress variation ($\sigma_z$) versus pressure applied to a piezoresistive pressure microsensor with a square-shaped diaphragm (1000 µm length) and considering thicknesses of 10, 15, and 20 µm.

Fig. 11. Output voltage variation of a Wheatstone bridge versus pressure applied to a piezoresistive pressure microsensor with a square-shaped diaphragm (1000 µm length) and considering thicknesses of 10, 15, and 20 µm.
The result of the maximum normal stresses (σₓ) in the diaphragm of the microsensor is shown in Fig. 10. This response was obtained using the polynomial model, the Timoshenko model and the FEM model. For a pressure lower than 20 kPa and a thickness greater than 10 µm, the maximum normal stress calculated using the polynomial model agrees well with the other two models. For the diaphragm 15 µm thick, the polynomial model showed a maximum stress of 45.37 MPa versus 55.17 and 49.60 MPa obtained with the Timoshenko model and the FEM model, respectively. These values are less than the rupture stress of ⟨100⟩ silicon [26]. Moreover, the diaphragm 10 µm thick exhibited the highest normal stress (102.1 MPa using the polynomial model and 124.1 MPa using the Timoshenko model).

The results of the normal stress components obtained by the three models (polynomial, Timoshenko and FEM models) were introduced into Eq. (34) to find the electric behavior of the Wheatstone bridge of the pressure microsensor. Figure 11 shows the output voltage of the Wheatstone bridge versus the pressure (0-40 kPa) applied to the diaphragm. In this case, a supply voltage (Vₘₚ) of 1 V and three different thicknesses for the diaphragm were considered. For the two thicknesses with higher magnitude (15 and 20 µm), the results obtained with the polynomial model agree well with the Timoshenko model and the FEM model. In addition, the results of sensitivity (mV/V/kPa) of the pressure microsensor are shown in Table I. The highest sensitivity was calculated for the diaphragm 10 µm thick. Using the polynomial model, the diaphragm 10 µm thick has a maximum sensitivity of 1.64 mV/V/kPa, while the two diaphragms that are 15 µm and 20 µm thick have sensitivities of 0.73 mV/V/kPa and 0.41 mV/V/kPa, respectively. Therefore, the electromechanical design of the piezoresistive pressure microsensor showed that the square-shaped diaphragm 1000 µm length and 10 µm thickness has an adequate sensitivity and a safe mechanical response for measuring blood pressure and heart rate in the pressure range from 0 to 40 kPa. This design was obtained using the proposed polynomial model with a lower computing time than the Timoshenko model and the FEM model.

4. Conclusions

A novel polynomial model was developed to predict the electromechanical behavior of piezoresistive pressure microsensors with square-shaped diaphragms more easily and quickly for low-pressure biomedical applications. This model was determined using the small-deflection theory for the bending of thin plates and the Ritz method. The expressions of the polynomial model are simpler than the classical Timoshenko model for plates. A pressure microsensor with a square-shaped diaphragm (1000 µm length) and with three different thicknesses (10, 20, and 30 µm) was studied. For small deflections of the diaphragm, the electromechanical behavior of the pressure microsensor obtained using the polynomial model agrees well with the Timoshenko model and the FEM model. The deflections of the diaphragm (10 µm thick) calculated using the polynomial model showed a relative difference of 5.14 and 0.92% with respect to the Timoshenko model and the FEM model, respectively. In addition, the deflections of the diaphragm exhibited a reduction of 70.38 and 87.50% for thicknesses of 15 and 20 µm, respectively. A maximum sensitivity (1.64 mV/V/kPa) was calculated for the diaphragm 10 µm thick.

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