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Keywords: Synchronization; noise; activation processes.

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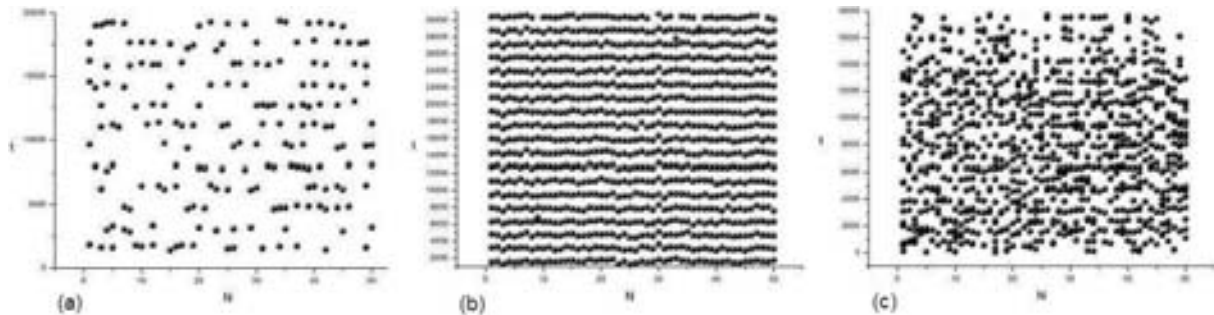


FIGURE 1. Transition times for the N subsystems, for $D= 0.02, 0.04$ and 0.07 from left to right.

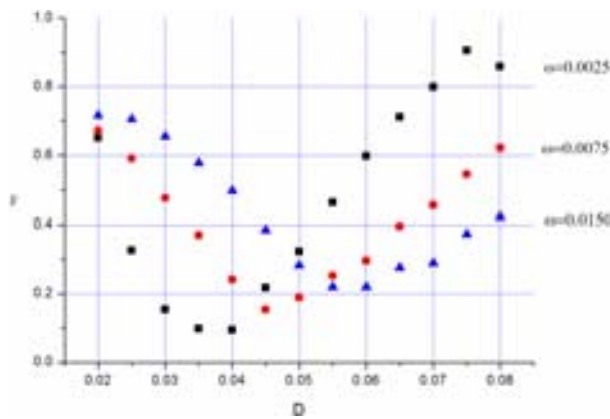


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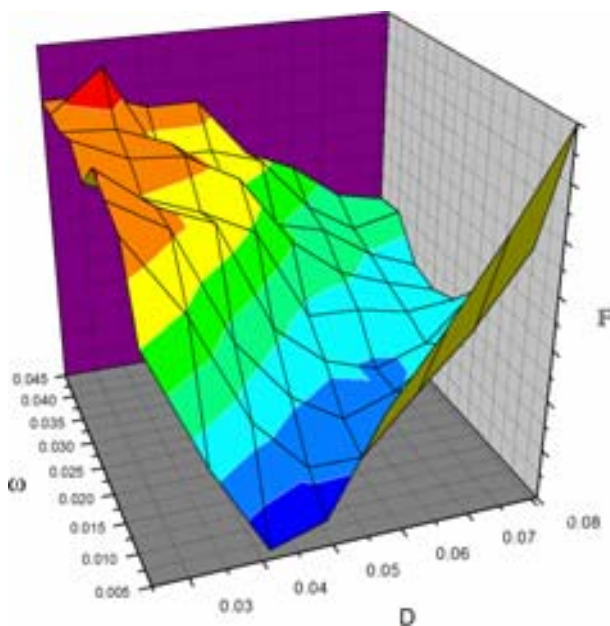


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In order to appreciate graphically that the noise may have a constructive effect on the synchronization of the set of bistable subsystems, we solved Eq.(1) numerically for $N = 50$ subsystems by means of a fourth-order Runge Kuta algorithm, using the same initial condition $x(0) = -1$ (the left side minimum) and plotted the sequence of transition times for each of the subsystems (all of them measured with respect to the same initial time). Here we fixed an external frequency $\omega = 0.005$, and then we repeated these calculations for three different values of D (0.02, 0.04, 0.07). In Fig. 1 we show the transition times of the N subsystems (plotted in N columns) for the three chosen values for D .

What we see in these plots shows that there is a value of D for which there is better synchronization of the N bistable subsystems, Fig. 1b. In these graphs we observe that the synchronization assumes a qualitative ordering effect. In order to quantify and characterize more precisely this synchronization phenomenon, it is necessary to define an adequate measure of the synchronization. Here we recall that the phenomenon of

stochastic resonance, very extensively studied in bistable systems [10, 11], is usually detected by measuring the *spectral amplification*, which in the linear-response regime is defined as [11]:

$$\eta(D, \omega) = \left[\frac{A(D, \omega)}{A_0} \right]^2. \tag{3}$$

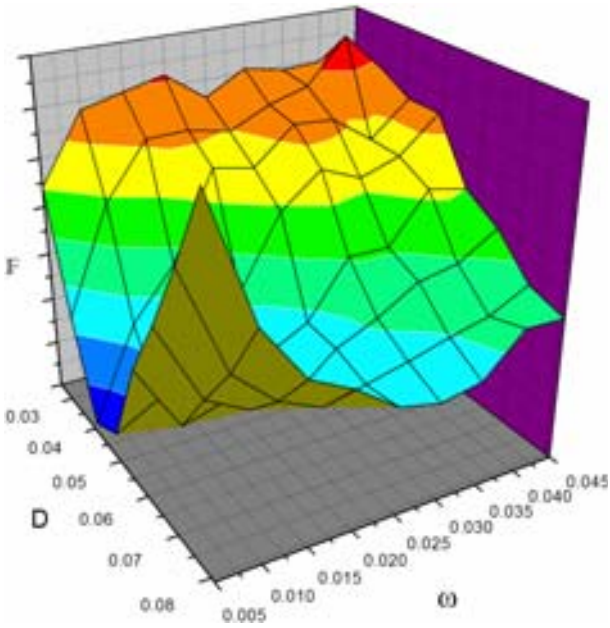


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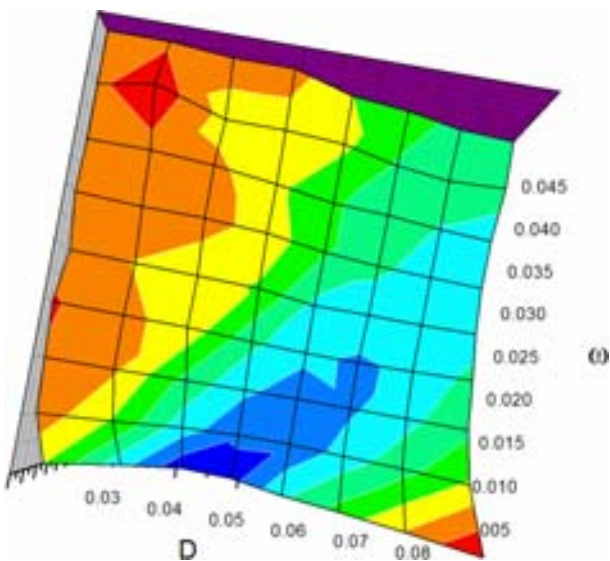


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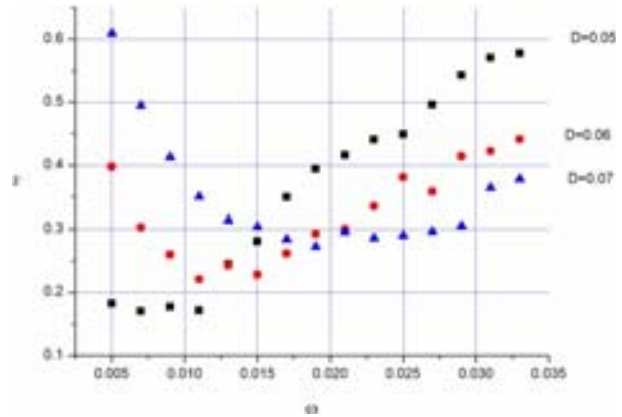


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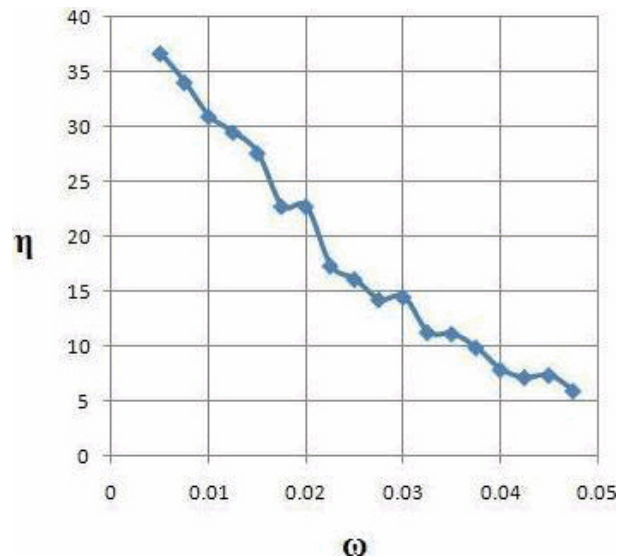


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Here A_0 is the amplitude of the periodic forcing, and $A(D, \omega)$ is the amplitude of the mean position of the particle $\langle x(t) \rangle = A \sin(\omega t + \phi_0)$, where the average is taken over an ensemble of noise realizations. However, if we are interested in the synchronization of the *transition times* of a set of bistable subsystems, the use of the above spectral amplification is not adequate, since this quantity might have significant non-zero values even when no transitions occur at all. However, a good measure of the synchronization of the transition times of a set of bistable elements is the *fractional fluctuation* (\mathbb{F}) of the sequence of *all* the transition times, t_{tr} , (of all the subsystems), which is defined as follows:

$$\mathbb{F} = \frac{\sqrt{\langle t_{tr}^2 \rangle - \langle t_{tr} \rangle^2}}{\langle t_{tr} \rangle}. \tag{4}$$

This quantity depends on the noise intensity and the frequency of the external field, and in Fig. 2 we can see the graphs of $\mathbb{F}(D)$ for three different values of ω .

This figure shows that for each value of ω , $\mathbb{F}(D)$ presents a well-defined minimum which corresponds to the optimal synchronization, and the position of the minimum depends on the frequency of the external field. In order to appreciate the dependence of \mathbb{F} on both parameters, D and ω , in Figs. 3 and 4 we present the surface $\mathbb{F}(D, \omega)$ as seen from two different angles. Fig. 5 shows the surface $\mathbb{F}(D, \omega)$ as seen from above, and therefore we can appreciate the shape of the level curves.

These figures clearly show that the intersection of the surface $\mathbb{F}(D, \omega)$ with a plane $\omega = \omega_0$ yields a curve $\mathbb{F}(D, \omega_0)$ with a clear minimum. In fact, in Fig. 5 we can see that a plane $\omega = \omega_0$, with $\omega_0 < 0.02$, intersects the lowest portion of the surface $\mathbb{F}(D, \omega)$. The existence of this minimum is the analog of the maximum of $\eta(D, \omega_0)$, which is found when the stochastic resonance phenomenon occurs in a single bistable system. On the other hand, if we intersect the surface $\mathbb{F}(D, \omega)$ with a plane $D = D_0$ we also obtain curves $\mathbb{F}(D_0, \omega)$ with well defined minima for some values of D_0 , thus enabling us to select the frequency which leads to the best synchronization of the set of bistable systems. In Fig. 6 we can appreciate the shape of three curves of this type.

It is important to emphasize that the existence of the minimum of the function $\mathbb{F}(D_0, \omega)$ has no counterpart when the function $\eta(D_0, \omega)$ is considered (see Fig. 7). Indeed, this function decreases monotonically with ω (when a bistable system is considered), and therefore, if we are interested in the synchronization of a set of bistable subsystems, the knowledge of $\eta(D, \omega)$ does not allow us to choose an optimal frequency for a given value of the noise intensity. This fact implies that the optimal conditions for the synchronization phenomenon studied in this work differs from the conditions which optimize the stochastic resonance of a single bistable system. As mentioned above, the fractional fluctuation $\mathbb{F}(D, \omega)$ is indeed an adequate measure to quantify the synchronization of the transition times of a set of bistable elements. The geometrical structure of the function $\mathbb{F}(D, \omega)$ clearly shows the region (within the space D - ω) where the synchronization is optimal, and therefore it contains useful information for controlling a collection of bistable subsystems.

In closing this letter, it is worth mentioning the following points. It is essential to emphasize that the 3D plots in Figs. 3, 4 and 5 show that, in order to achieve good synchronization, an interplay between D and ω should be maintained. That is, if ω increases, D should also be increased. With regard to the influence of the noise intensity, we can appreciate in Fig. 1 that for very small values of D the transitions are scarce and not coupled to the frequency. Therefore, synchronization is not favoured in this case. We can also see that for high values of D , the transitions occur very frequently since the noise dominates the transitions and the synchronization also becomes very poor. However, for intermediate values of D the system tends to the linear response regime (the rows that appear in Fig. 1b are an average of one period apart) and it is here that the synchronization can be enhanced depending, as mentioned above, on the frequency values.

Here we assumed that the interaction between the bistable subsystems was negligible in comparison to the effects of the external force and the noise; obviously this restricts the usefulness and predictive value of the information given by $\mathbb{F}(D, \omega)$. Therefore, it would be important to study how this function is modified when different interaction mechanisms are taken into account. It should also be pointed out that in the present letter we have considered the effect of white noise only. The effect of colored noise is significantly more complicated since it incorporates an additional parameter, the correlation time of the noise, and this study is in progress. As a final remark, we would like to point out that it would be interesting to investigate how the present analysis should be modified if the *non-inertial* assumption is removed. Although the behavior of a set of bistable subsystems with inertia might have some similarities when colored noise is considered, this relationship merits further investigation.

Acknowledgments

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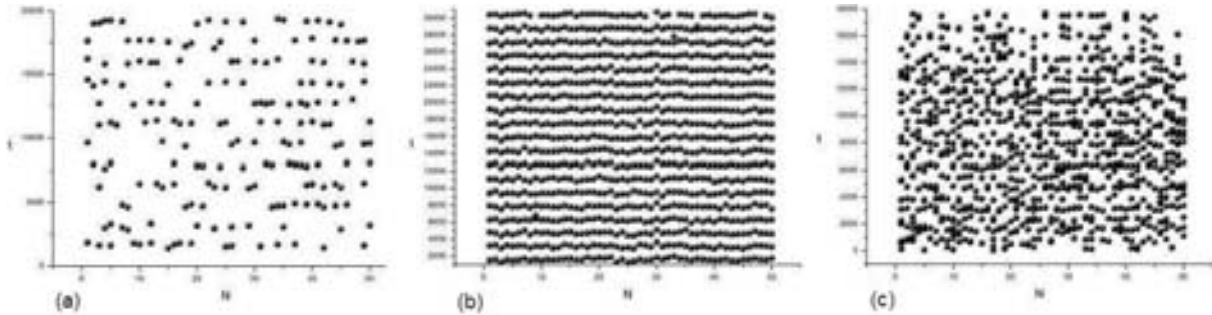


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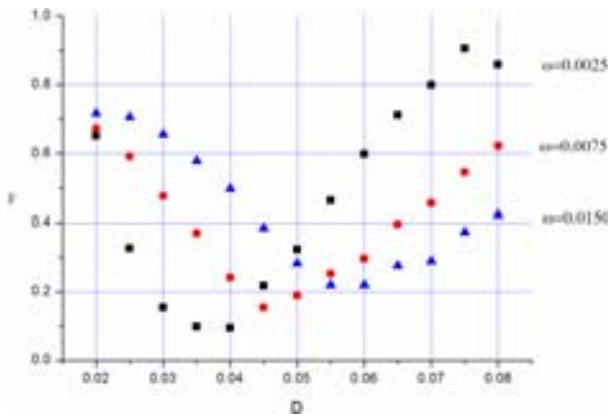


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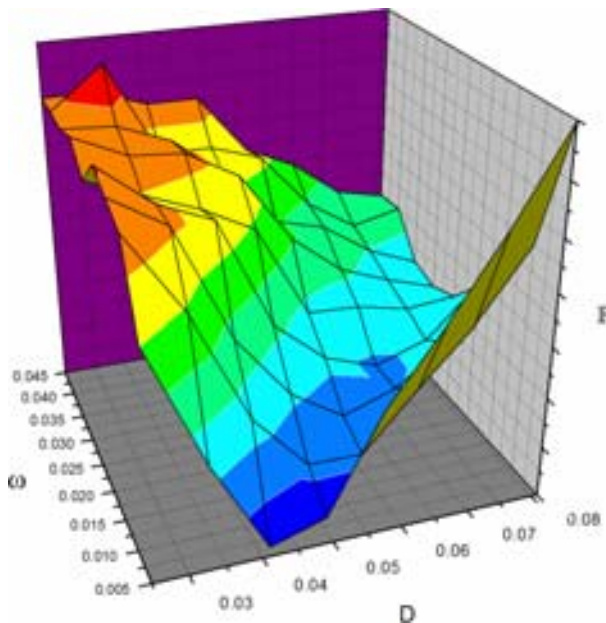


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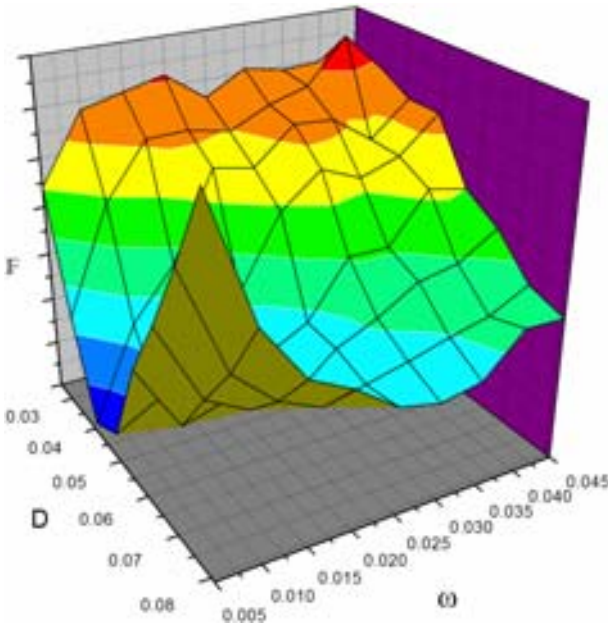


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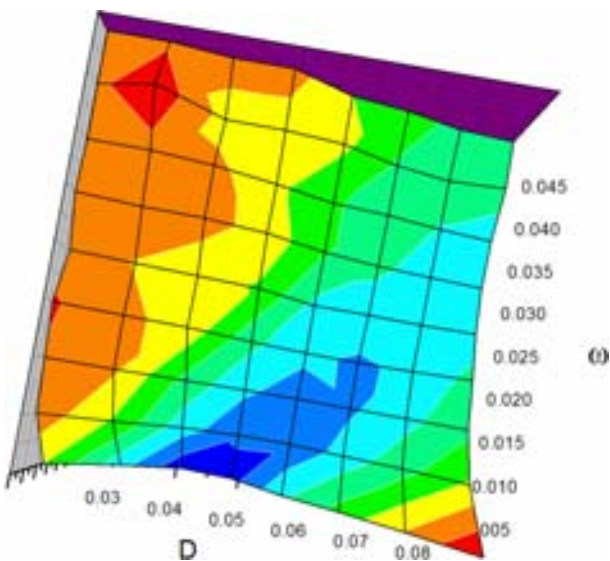


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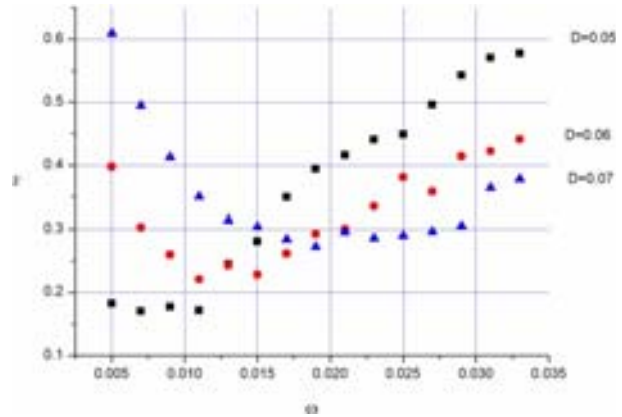


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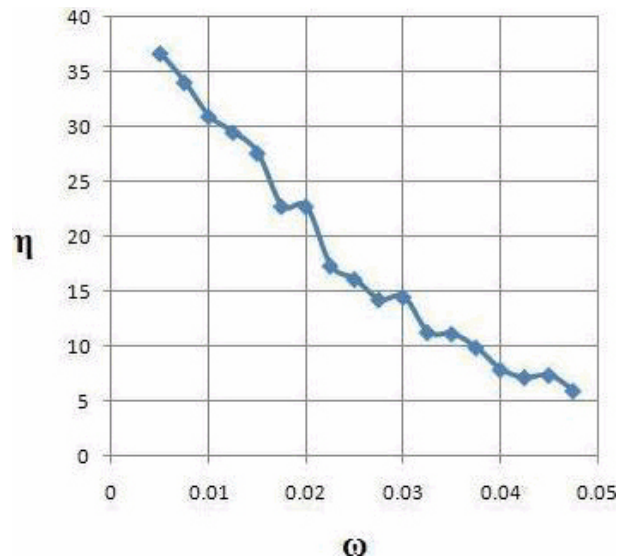


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It is important to emphasize that the existence of the minimum of the function $\mathbb{F}(D_0, \omega)$ has no counterpart when the function $\eta(D_0, \omega)$ is considered (see Fig. 7). Indeed, this function decreases monotonically with ω (when a bistable system is considered), and therefore, if we are interested in the synchronization of a set of bistable subsystems, the knowledge of $\eta(D, \omega)$ does not allow us to choose an optimal frequency for a given value of the noise intensity. This fact implies that the optimal conditions for the synchronization phenomenon studied in this work differs from the conditions which optimize the stochastic resonance of a single bistable system. As mentioned above, the fractional fluctuation $\mathbb{F}(D, \omega)$ is indeed an adequate measure to quantify the synchronization of the transition times of a set of bistable elements. The geometrical structure of the function $\mathbb{F}(D, \omega)$ clearly shows the region (within the space D - ω) where the synchronization is optimal, and therefore it contains useful information for controlling a collection of bistable subsystems.

In closing this letter, it is worth mentioning the following points. It is essential to emphasize that the 3D plots in Figs. 3, 4 and 5 show that, in order to achieve good synchronization, an interplay between D and ω should be maintained. That is, if ω increases, D should also be increased. With regard to the influence of the noise intensity, we can appreciate in Fig. 1 that for very small values of D the transitions are scarce and not coupled to the frequency. Therefore, synchronization is not favoured in this case. We can also see that for high values of D , the transitions occur very frequently since the noise dominates the transitions and the synchronization also becomes very poor. However, for intermediate values of D the system tends to the linear response regime (the rows that appear in Fig. 1b are an average of one period apart) and it is here that the synchronization can be enhanced depending, as mentioned above, on the frequency values.

Here we assumed that the interaction between the bistable subsystems was negligible in comparison to the effects of the external force and the noise; obviously this restricts the usefulness and predictive value of the information given by $\mathbb{F}(D, \omega)$. Therefore, it would be important to study how this function is modified when different interaction mechanisms are taken into account. It should also be pointed out that in the present letter we have considered the effect of white noise only. The effect of colored noise is significantly more complicated since it incorporates an additional parameter, the correlation time of the noise, and this study is in progress. As a final remark, we would like to point out that it would be interesting to investigate how the present analysis should be modified if the *non-inertial* assumption is removed. Although the behavior of a set of bistable subsystems with inertia might have some similarities when colored noise is considered, this relationship merits further investigation.

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