Radiative generation of light fermion masses in a $SU(3)_H$ horizontal symmetry model

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In a model with a gauge group $SU(3)_H \otimes G_{SM}$, where $SU(3)_H$ is a horizontal symmetry and $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the standard model, we propose a radiative mechanism of mass generation mediated by the $SU(3)_H$ gauge bosons for the light fermions, meanwhile the masses of the heaviest family are generated by the implementation of seesaw mechanisms with the introduction of vectorial fermions.

Keywords: Horizontal symmetry; fermion masses.

1. Introduction

The known quark and lepton masses are generated after the spontaneous breaking of the electroweak symmetry to $U(1)_Q$ of Quantum Electrodynamics. However, the responsible mechanism of this symmetry breaking as well as the generation mechanism of the fermion masses, including their origin and hierarchy, remain unknown, and they are two of the current puzzles of the elementary particle physics. In the literature there are many proposals trying to explain the mass hierarchy, the fermion mixing, and their possible relation to new physics [1].

A possible answer to why the masses of the light fermions are so small compared with the electroweak scale is that they arise through radiative corrections [2], while the mass of the top quark, and probably those of the bottom quark, and of the tau lepton are generated at tree level. This may be understood as a consequence of the breaking of a symmetry among families (a horizontal symmetry). This symmetry may be discrete [3], or continuous, [4]. The radiative generation of light fermions may be mediated by scalar particles as it is proposed, for instance, in references [2] and [5] or also through vectorial bosons as it happens in “Dynamical Symmetry Breaking” (DSB) theories like “Extended Technicolor” [6].

In this work we propose a mechanism in which the light fermions get mass through loop diagrams mediated by the massive vectorial bosons of a horizontal symmetry that is spontaneously broken, whereas the masses of the top and bottom quarks, as well as the tau lepton are generated by the implementation of seesaw mechanisms with the introduction of new fermions of vectorial type.

2. The model

We define the gauge group symmetry

$$G \equiv SU(3)_H \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y,$$

where $SU(3)_H$ is a horizontal symmetry among families and

$$G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

is the gauge group of the “Standard Model” of elementary particles. The content of fermions assumes the ordinary quarks and leptons including the sector of right-handed neutrinos, assigned under the $G$ group in the form:

$$\Psi_q = \begin{pmatrix} 3, 3, 2, 1 \over 3 \end{pmatrix}_L$$ (1)

$$\Psi_u = \begin{pmatrix} 3, 3, 1, 4 \over 3 \end{pmatrix}_R$$ (2)

$$\Psi_d = \begin{pmatrix} 3, 3, 1, -2 \over 3 \end{pmatrix}_R$$ (3)

$$\Psi_l = \begin{pmatrix} 3, 1, 2, -1 \end{pmatrix}_L$$ (4)

$$\Psi_\nu = \begin{pmatrix} 3, 1, 1, 0 \end{pmatrix}_R$$ (5)

$$\Psi_e = \begin{pmatrix} 3, 1, 1, -2 \end{pmatrix}_R$$ (6)

where the last entry correspond to the hypercharge ($Y$) and the electric charge is defined by $Q = T_{3L} + \frac{1}{2} Y$. The model also includes the introduction of the following vectorial fermions:
we introduce the scalar field:

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gauge bosons it is necessary to introduce more scalar fields.

fore, to complete the breaking of the horizontal symmetry
to be achieved in the form:

The “Spontaneous Symmetry Breaking” (SSB) is proposed
for known low energy physics, where \( \Lambda \)
so that the model has the possibility to be consistent with the
mass of the \( W \) charged gauge boson are:

contribution of \( \langle \Phi \rangle \) and \( \langle \Phi' \rangle \) to the
mass terms of the ordinary fermions and the vectorial
fermions. After the definition of the gauge group \( G \) and the
introduction of the right-handed neutrinos and the vectorial
fermions, under the standard model group, in the most simply non triv-
ial way under the horizontal symmetry, the introduction of the
right-handed neutrinos becomes a necessity to cancel anomal-
ies, while the introduction of the vectorial fermions has its
origin in the procedure to give masses at tree level only to the
heaviest family of known fermions. We consider that these
vectorial fermions play a crucial role to implement our
proposed hierarchical mass generation mechanism.

3. Symmetry breaking

The “Spontaneous Symmetry Breaking” (SSB) is proposed
to be achieved in the form:

\[ G \xrightarrow{\Lambda_1} G_{SM} \xrightarrow{\Lambda_2} SU(3)_C \otimes U(1)_Q \] (11)

so that the model has the possibility to be consistent with the
known low energy physics, where \( \Lambda_1 \) and \( \Lambda_2 \) are the scales
of SSB.

With the intention of implementing the mass generation mechanism for the ordinary quarks and leptons, and simulta-
neously to contribute to the SSB of \( SU(3)_H \) at the first stage,
we introduce the scalar field:

\[ \Phi^\prime = (3, 1, 1, 0), \] (12)

with ”Vacuum Expectation Value” (VEV),

\[ \langle \Phi^\prime \rangle^T = (0, 0, v') \] (13)

where \( T \) means transpose. This scalar \( \Phi^\prime \) with the above
VEV generates the breaking of \( SU(3)_H \) to \( SU(2)_H \). Therefore,
to complete the breaking of the horizontal symmetry and to produce the appropriate mixing among their neutral
gauge bosons it is necessary to introduce more scalar fields.
The details to achieve this goal are not treated in this pa-
per. However, we can mention here that the additional Higgs scalars should not be in the fundamental representation of
\( SU(3)_H \) so that they do not spoil the desired hierarchical
fermion mass generation mechanism.

To achieve the spontaneous breaking of the electroweak
symmetry to \( U(1)_{Q} \), we introduce the scalars:

\[ \Phi = (3, 1, 2, -1) \] (14)

\[ \Phi^\prime = (3, 1, 2, +1) \] (15)

with the VEV’s

\[ \langle \Phi \rangle^T = (\langle \Phi_1 \rangle, \langle \Phi_2 \rangle, \langle \Phi_3 \rangle) ; \] \( \langle \Phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_i \\ 0 \end{pmatrix} \) \] (16)

\[ \langle \Phi' \rangle^T = (\langle \Phi'_1 \rangle, \langle \Phi'_2 \rangle, \langle \Phi'_3 \rangle) ; \] \( \langle \Phi'_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ V_i \end{pmatrix} \) \] (17)

where \( i = 1, 2, 3 \). The contributions of \( \langle \Phi \rangle \) and \( \langle \Phi' \rangle \) to the
mass of the \( W \) charged gauge boson are:

contribution of \( \langle \Phi \rangle \):

\[ \frac{1}{2} g (v_1^2 + v_2^2 + v_3^2) \frac{1}{2} \] (18)

contribution of \( \langle \Phi' \rangle \):

\[ \frac{1}{2} g (V_1^2 + V_2^2 + V_3^2) \frac{1}{2} \] (19)

where \( g \) is the \( SU(2)_L \) coupling constant. If we take into
account now that the horizontal symmetry has already been
completely broken at the first stage and if \( \Lambda_1 \gg \Lambda_2 \), then
there are not arguments of symmetry to align the VEV’s of \( \Phi \)
and \( \Phi' \) under \( SU(3)_H \), that is, we can assume for simplicity that

\[ v_1 = v_2 = v_3 = V_1 = V_2 = V_3 = \frac{v}{2 \sqrt{6}} \] (20)

where \( v \approx 246 \) GeV and then \( M_W = (1/2)gv \). Note
that \( \Phi \) and \( \Phi' \) transform as triplets under \( SU(3)_H \). This
fact has as consequences that they contribute slightly to the
\( SU(3)_H \) gauge boson masses and produce mixing between these
bosons with the neutral boson \( Z \) of the standard model.
The consequences of this phenomenon will not be discussed in
this report.

4. Fermion masses

Now, we describe briefly how to obtain the mass terms of the
ordinary fermions. The analysis is presented for the up-quark
sector, with a completely analogous procedure for the other
sectors.

With the fields of particles introduced in the model, we
may write the following gauge invariant Yukawa couplings:

\[ h_u \bar{u}_R \Phi U_R + h' \bar{u}_u \Phi^\prime U_L + M \bar{U}_L U_R + h.c \] (21)

where \( h_u \) and \( h' \) are Yukawa coupling constants.

When \( \Phi \) and \( \Phi^\prime \) acquire VEV’s we obtain the mass terms

\[ \bar{u}_L M_{u} \Phi^\prime_{R} + h.c \] (22)

where:
\[ \Psi_L^T = (u_L^o, c_L^o, t_L^o, U_L) \]  
\[ \Psi_R^T = (u_R^o, c_R^o, t_R^o, U_R) \]

with \( h = (1/\sqrt{2}) h_u \). When this mass matrix \( M_u^o \) is diagonalized by a biunitary transformation, we find to this order two associated zero eigenvalues to the masses of the light up quarks, while the two eigenvalues different from zero are associated with the masses of the top quark and the vectorial fermion \( U \).

Explicitly, if we define

\[ \Psi_L^o = V_L^o \Psi_L \]
\[ \Psi_R^o = V_R^o \Psi_R \]

where

\[ \lambda_\pm = \frac{1}{2} \left( B \pm \sqrt{B^2 - 4D} \right) \]

\[ B = 3a^2 + b^2 + c^2 \]
\[ D = 3a^2b^2 \]

with

\[ a \equiv h v \quad , \quad b \equiv h' v' \quad , \quad c \equiv M \]

the orthogonal matrices \( V_L^o \) and \( V_R^o \) become

\[ V_L^o = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \alpha & \frac{1}{\sqrt{2}} \sin \alpha \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \alpha & \frac{1}{\sqrt{2}} \sin \alpha \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \alpha & \frac{1}{\sqrt{2}} \sin \alpha \\ 0 & 0 & -\sin \alpha & \cos \alpha \end{pmatrix} \]

where

\[ \cos \alpha = \frac{(\lambda_+ - 3a^2)}{\sqrt{3a^2b^2 + (\lambda_+ - 3a^2)^2}} \]
\[ \sin \alpha = \frac{\sqrt{3ac}}{\sqrt{3a^2b^2 + (\lambda_+ - 3a^2)^2}} \]

and

\[ V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \beta & \sin \beta \\ 0 & 0 & -\sin \beta & \cos \beta \end{pmatrix} \]

with

\[ \cos \beta = \frac{(\lambda_+ - b^2)}{\sqrt{b^2c^2 + (\lambda_+ - b^2)^2}} \]
\[ \sin \beta = \frac{bc}{\sqrt{b^2c^2 + (\lambda_+ - b^2)^2}} \]

From these matrices \( V_L^o \) and \( V_R^o \) we obtain

\[ V_L^o T M_u^o V_R^o = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\lambda_-} & 0 \\ 0 & 0 & 0 & \sqrt{\lambda_-} \end{pmatrix} \]

and the top-quark mass is given approximately by

\[ m_t \approx \sqrt{3h} \frac{v \nu'}{M_U} \]

\( M_U \) being the mass of the vectorial quark \( U \).

Subsequently the masses of the light up quarks arise through one loop diagrams. After the breakdown of the electroweak symmetry, we can construct the generic one and two loop mass diagram of Fig. 1. The vertices in this diagram come from the gauge interaction Lagrangian

\[ i L_{int} = \frac{g_H Y}{\sqrt{2}} \left( \bar{u}_{kL} \gamma_{\mu} u_{jL} Y_{ij}^\mu + \bar{u}_{R} \gamma_{\mu} t_{R} Y_{ij}^\mu \right) + L \leftrightarrow R + h.c \]

\( g_H \) (\( Y \)) being the \( SU(3)_H \) coupling constant (gauge bosons), \( i, j, k = 1, 2, 3 \) denote family indices, the crosses in the internal fermion line means the mixings, and the mass \( M \), generated by the couplings of Eq. (21) after \( \Phi \) and \( \Phi'' \) take VEV’s and the black circle in the boson line means the tree level mixing mass term

\[ M_{ij} = \frac{M^2 Y_{ik} Y_{lj}}{M_\nu} \]

generated in the symmetry breaking process.
5. Discussion

At present we are performing a more complete study of the model introduced in this report, including some phenomenology and confrontation with the experimental data. An important aspect to study for the last, is the calculation of some processes of “Flavor Changing Neutral Currents” (FCNC) that can be induced in the model as a consequence of the particles introduced, for instance the gauge bosons of the $SU(3)_H$ horizontal symmetry. Some examples of these FCNC processes are the radiative “lepton flavor violation” (LFV) decays: $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$, and $\tau \rightarrow \mu\gamma$, which are induced by the radiative mass generation mechanism when we attach a photon to the internal charged leptonic lines in the one loop mass diagrams, and $\mu \rightarrow eee$, $\tau \rightarrow \mu\mu\mu$, $\tau \rightarrow \mu\mu\epsilon$, $\tau \rightarrow \epsilon\epsilon\epsilon$, coming from tree level diagrams like the one in Fig. 2. The calculation involved, including the muon anomalous magnetic moment induced as well by the radiative muon mass generation mechanism [7], is in progress and will be reported elsewhere.

\[ M_U \simeq \sqrt{3}h h' \frac{v'}{m_t} \]

If in last equation we consider the values $m_t \simeq 175 \text{ GeV}$ and $v \simeq 246 \text{ GeV}$, then by assuming $\sqrt{3}h h' \simeq (m_t/v) \simeq 0.711$ we could expect $M_U$ to be of the same order of $v'$, that is, of the order of the scale of SSB of $SU(3)_H$ that could work in the $\text{TeV}$ region. This order of magnitude may also be expected for the other vectorial fermion masses if one supposes that the corresponding products of Yukawa couplings behave like $\sqrt{3}h d_h' \simeq (m_h/v)$ and $\sqrt{3}h u' \simeq (m_t/v)$. However, the actual values of the Yukawa coupling constants, the VEV $v'$, and the masses of the vectorial fermions involved should be determined by demanding consistency between the fermion masses and mixing angles with the adequate suppression of FCNC (if this suppression is realized).

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