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An echo of an exciting light pulse in quantum wells

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It is shown that non-sinusoidal oscillations appear in transmitted, reflected and absorbed light fluxes when light pulses irradiate a semiconductor quantum well (QW) containing a large number of equidistant energy levels of electronic excitations. The oscillation amplitude is comparable to the flux values for short pulses with the duration $\gamma_l^{-1} \leq \hbar/\Delta E$. A damped echo of the exciting pulse appears in the time intervals $2\pi\hbar/\Delta E$ for very short light pulses $\gamma_l^{-1} \ll \hbar/\Delta E$. Symmetric and asymmetric pulses with a sharp front are considered. Our theory is applicable to narrow QW's in a quantizing magnetic field when the equidistant energy levels correspond to the electron-hole pairs with different Landau quantum numbers.

Keywords: Quantum wells; light absorption and transmission.

Se demuestra que aparecen oscilaciones no sinusoidales en los flujos de luz que se transmiten, reflejan y absorben cuando pulsos de luz irradian un pozo cuántico semiconductor (QW), que contiene un gran número de niveles equidistantes de energías de excitaciones electrónicas. La amplitud de la oscilación es comparable a los valores de flujo de pulsos cortos, cuya duración es $\gamma_l^{-1} \leq \hbar/\Delta E$. Un eco amortiguado del pulso excitante aparece en los intervalos de tiempo $2\pi\hbar/\Delta E$ para el caso de pulsos de luz muy cortos $\gamma_l^{-1} \ll \hbar/\Delta E$. Se consideran pulsos simétricos y asimétricos con un frente agudo. Nuestra teoría se aplica para QWs estrechos en un campo magnético cuantizante, cuando los niveles de energía equidistantes corresponden a pares electrón-hueco con diferentes números cuánticos de Landau.

Descriptores: Pozos cuánticos; absorción y transmisión de luz.

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1. Introduction

A large number of theoretical and experimental investigations is devoted to the elaborate study of electronic excitations in bulk crystals and semiconductor QW's with the help of time resolved scattering (TRS), because the existence of discrete energy levels produces the most interesting results obtainable by the TRS. It is well known that two closely disposed energy levels demonstrate a new effect: sinusoidal beats appear in reflected and transmitted pulses with a frequency corresponding to the energy distance between energy levels [1].

We investigate, theoretically, the reflection and absorption of light pulses irradiating semiconductor QWs with a large number of equidistant energy levels. We suppose that the light pulse carrier frequency ω_l is in resonance with one of the energy levels.

Two variants are now possible: first, the influence of the remaining energy levels may be discarded; second, a few of the neighboring energy levels must be taken into account. The first variant is realized, for instance, by a symmetric pulse under the condition $\gamma_l \ll \Delta\omega$, where $\Delta\omega$ is the distance between the neighboring levels, and γ_l is the parameter characterizing the pulse duration. It is shown [2] that under the condition $\gamma_r \geq \gamma$, where $\gamma_r(\gamma)$ is the radiative (non-radiative) broadening of an electronic excitation, the transmitted pulse profile changes drastically. The lifetimes of the electron-hole pairs (EHP) for a QW in a quantizing magnetic field are calculated in [3, 4]. The second variant, with various equidistant energy levels, is considered in [3, 5], where a ladder-like structure of reflected and transmitted pulses is predicted.

We consider both symmetric and asymmetric pulses, giving the results for different γ_l, and showing that very short pulses create ”an echo” in reflected and transmitted light.

2. Theory

Let us now consider a QW with a system of equidistant energy levels

$$\omega_\rho = \rho\Omega_\mu, \qquad \rho = 0, 1, 2\cdots \tag{1}$$

The real significance of the designation Ω_μ is given below. We assume that the QW's width is much smaller than the light wavelength $d \ll c/n\omega_l$, where n is the refraction index outside the QW. The excitation ρ is characterized by the radiative $\gamma_{r\rho}$ and non-radiative broadenings.

We suppose that the light pulse reaches the QW's plane xy perpendicularly from the left (from the negative z)

and its electric field is

$$\mathbf{E}_0(z,t) = E_0 \mathbf{e}_l e^{-i\omega_l p}\{\Theta(p)e^{-\gamma_{l1}p/2} \\ +[1-\Theta(p)]e^{\gamma_{l2}p/2}\} + c.c., \qquad (2)$$

where E_0 is the real amplitude, $\mathbf{e}_l$ is the circular polarization vector, $p = zn/c,$ and $\Theta(p)$ is the Haeviside function. For a symmetric pulse, we have

$$\gamma_{l1} = \gamma_{l2} = \gamma_l, \qquad (3)$$

and for the pulse with a sharp front

$$\gamma_{l1} = \gamma l, \quad \gamma_{l2} \to \infty. \qquad (4)$$

The electric fields on the left and right of the QW and corresponding time-dependent energy fluxes are calculated according to the methods in [2–6]. We demonstrate results obtained for the following parameters

$$\gamma_{r\rho} = \gamma_r, \quad \gamma_\rho = \gamma, \quad \gamma_r \ll \gamma \ll \gamma_l, \quad \gamma \ll \Omega_\mu, \qquad (5)$$

and for an arbitrary interrelation between γ_l and Ω_μ. We consider the energy fluxes under the conditions

$$p \gg \gamma_l^{-1}, \quad s \gg \gamma_l^{-1}, \quad p \ll \gamma_r^{-1}, \quad s \ll \gamma_r^{-1}, \qquad (6)$$

where $s = t + zn/c$ is the variable for the reflected energy flux. Consequently, the contributions to the electric fields containing the factors $\exp(-\gamma_l p/2)$ or $\exp(-\gamma_l s/2)$ become insignificant. We find that only those contributions in induced fields which are proportional to $\exp(-\gamma p/2)$ or $\exp(-\gamma s/2)$ are essential. The existence of such contributions is proved in many investigations. The induced fields outside the QW are symmetric, as only narrow QWs are considered. It follows from this fact that the transmitted and reflected fluxes are equal in magnitude, and the absorbed flux is equal to twice the transmitted (or reflected) flux with the opposite sign. A negative absorption indicates that the QW releases stored energy, radiating it symmetrically with two fluxes on the left and on the right.

The reflected energy flux results in

$$\mathcal{R}(s) = 4(\gamma_r/\gamma_l)^2 e^{-\gamma s} Y_{\Omega_l,G_l}(S), \qquad (7)$$

where $\mathcal{R}(s)$ is the magnitude of the reflected flux in units $cE_0^2/(2\pi n)$, and Y_{Ω_l,G_l} is the dimensionless periodical function of the variable $S = \Omega_\mu s$ with the period 2π dependent on the parameters

$$\Omega_l = \omega_l/\Omega_\mu, \qquad G_l = \gamma_l/\Omega_l. \qquad (8)$$

The frequency ω_l is counted from the same level as Ω_μ. The factor $\exp(-\gamma s)$ determines the damping of the reflected flux. The function Y is periodical on s with $2\pi/\Omega_\mu$. These oscillations are not sinusoidal.

3. Results

We calculate the functions $Y_{\Omega_l,G_l}(S)$, and show that the results are essentially dependent on the parameter G_l. For $G_l \ll 1$, the periodical vibrations of the intensities of the reflected and transmitted energy fluxes are very small in amplitude. The values $\Omega_l = 0, 1, 2\ldots$ correspond to the resonance of the frequency ω_l with one of the energy levels. A slight detuning of the frequency ω_l results in a drastic reduction of the energy fluxes.

The function $Y^{sym}_{\Omega_l,G_l}(S)$ inside of one period is shown in Figs. 1, 2.

The upper index indicates an exciting symmetric pulse. The function $Y^{sym}_{\Omega_l,G_l}(S)$ is relatively symmetric to the substitution S by $2\pi - S$. Figure 1 corresponds to the value G_l and to the set $\Omega_l = 0; 0.1; 0.5; 1; 1.5; 2$. The vibration amplitude in Fig. 1 reaches the value 1.5-2. The curves touch the abscissas axis in the point $S = \pi$ for $\Omega_l = 0.5$, and $\Omega_l = 1.5$ (*i.e.* when the frequency is disposed between the levels $\rho = 0$ and ¨ $\rho = 1$, and between the levels $\rho = 1$ and $\rho = 2$, respectively).

Figure 2 shows the echo of the exciting symmetric pulse. It corresponds to the large value $G_l = 5$ and to the set $\Omega_l = 0; 0.1; 0.5; 1$. At the points $s = 0$ and $s = 2\pi$, the function $Y^{sym}(S)$ increases sharply (reaching 30) when compared to corresponding values in the Fig. 1. However, they are very small at the intervals $s \gg G_l^{-1}$ and $(2\pi - S) \gg G_l^{-1}$. Thus, the periodical function $Y^{sym}(S)$ represents a sequence of short pulses whose duration is of the order of G_l^{-1}, and disposed with the intervals 2π. We apply Eq. (7) and find that at $\gamma_l \gg \Omega_\mu$ some echo of the exciting pulse has to be seen in the reflected energy flux, damping as $\exp(-\gamma s)$ in the interval $2\pi/\Omega_\mu$. This echo must also be present in the transmitted energy flux. Our results show that in the region of the Ω_l values, from 0 up to several units, the pulse replica form repeats.In a resonance ω_l

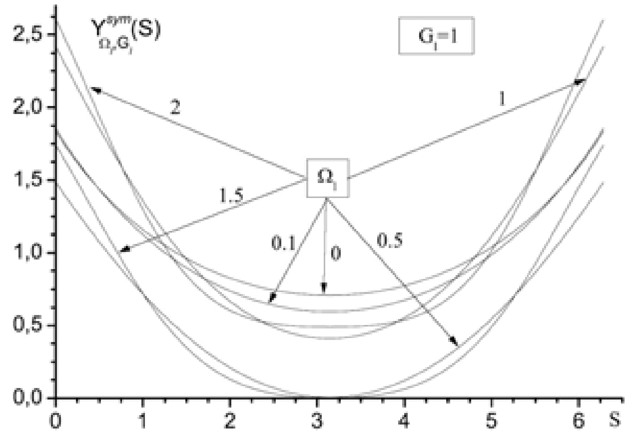

FIGURE 1. The function $Y^{sim}_{\Omega_l,G_l}(S)$, corresponding to the periodical factor in the value of the reflected energy flux when a QW is irradiated by the symmetric light pulse. The pulse duration $\gamma_l^{-1} = \Omega_\mu^{-1}$.

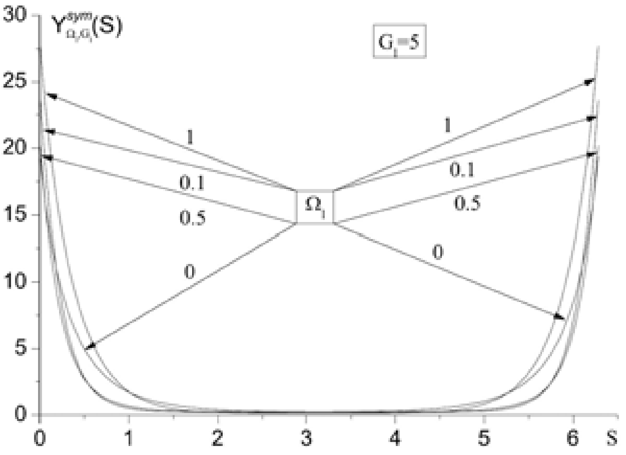

FIGURE 2. Same as Fig. 1 for the very small value γ_l^{-1}, when the echo of the exciting pulse appears.

with one of the upper levels with index $\rho \gg 1$, the form of the repeating pulses (echo) coincides with the exciting pulse form.

The repeating pulses contain the small factor

$$\delta = \pi^2(\gamma_r/\Omega_\mu)^2 e^{-\gamma s}. \tag{9}$$

This theory is applicable to narrow QWs in a quantizing magnetic field perpendicular to the QW's plane xy. The equidistant levels then correspond to the EHP with different Landau levels and fixed size quantized energy levels of electrons (holes). $\Omega_\mu = |e|H/\mu c$ is the cyclotron frequency, corresponding to the reduced mass $\mu = m_e m_h/(m_e + m_h)$, where $m_e(m_h)$ is the electron (hole) effective mass. For narrow QWs and quantizing magnetic fields, the energy level equidistance is small if

$$d \ll a, \qquad a_H \ll a, \tag{10}$$

where a is the radius of the Wannier-Mott exciton in a zero magnetic field, and $a_H = (c\hbar/(|e|H))^{1/2}$ is the magnetic length. We suppose that the band's non-parabolicity is small inside the vicinity of the band's extremes.

4. Conclusion

The dependence on time of reflected and absorbed fluxes of energy arising at normal irradiation, by an exciting light pulse on a quantum well placed in a quantizing magnetic field is investigated. The system of electronic excited energy levels in a quantum well was assumed equidistant, which is true in the case of quantizing magnetic fields and narrow quantum wells where it is possible to discard the Coulomb interaction between electrons and holes. After the exciting pulse has damped, transmitted and reflected pulses are equal in absolute values. The absorbed flux is negative and is equal in magnitude to twice the transmitted (or reflected) flux. Results for long, short and very short pulses are obtained. A damped echo of the exciting pulse in very short pulses is observed. When the carrier frequency of a pulse coincides with one of the very high energy levels in the quantum well, the form of repeating pulses coincides with that of the exciting pulse, which may be either symmetric or asymmetric.

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