A non-relativistic equation for spin-3/2 particles is proposed and the gyromagnetic ratio for charged spin-3/2 particles is determined.

Keywords: Spin-3/2 particles; gyromagnetic ratio.

Se propone una ecuación no relativista para partículas de espín 3/2 y se determina la razón giromagnética para partículas de espín 3/2 cargadas.

Descriptores: Partículas de espín 3/2; razón giromagnética.

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1. Introduction

In the quantum-mechanical description of particles, there are various, relativistic or non-relativistic wave equations whose form depends on the spin of the particles. The usual Schrödinger equation applies to the spin-0 particles in the non-relativistic domain, while the Klein–Gordon equation is the relativistic equation appropriate for spin-0 particles. The spin-1/2 particles are governed by the relativistic Dirac equation and the charge to mass ratio has a common value for spin-3/2 fields (see, e.g., Refs. 7 and the references cited therein), but we are not studying their non-relativistic limits.

In Sec. 2 the Schrödinger–Pauli equation for spin-1/2 particles is written making use of the Pauli matrices and of the two-component spinor notation which is employed in Sec. 3 to write the proposed equation for spin-3/2 particles. The notation and conventions used throughout this paper are summarized in Sec. 2; further details can be found in Refs. 8, 9.

2. Spin-1/2 particles

The usual Schrödinger equation for a spin-0 particle of mass \( M \) in a potential \( V(\mathbf{r}) \),

\[
-\frac{\hbar^2}{2M}\nabla^2 \psi + V(\mathbf{r})\psi = i\hbar \frac{\partial \psi}{\partial t},
\]

(1)
can be obtained from the classical Hamiltonian \( H = p^2 / 2M + V \), using the fact that the momentum operator in the coordinate representation is given by \(-i\hbar \nabla\). In the case of a spin-1/2 particle, the wave function is not a complex-valued function but a two-component spinor

\[
\psi(\mathbf{r}, t) = \begin{pmatrix} \psi_1(\mathbf{r}, t) \\ \psi_2(\mathbf{r}, t) \end{pmatrix},
\]

(2)

which under a rotation through an angle \( \alpha \) about the axis defined by a unit vector \( \mathbf{n} \) transforms into (see, e.g., Refs. 10, 9)

\[
\psi' = (\cos \frac{1}{2} \alpha I - i \sin \frac{1}{2} \alpha \mathbf{n} \cdot \mathbf{\sigma}) \psi,
\]

(3)

where \( I \) is the identity \( 2 \times 2 \) matrix and \( \mathbf{\sigma} = (\sigma_1, \sigma_2, \sigma_3) \) is formed by the Pauli matrices

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\]

\[
\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},
\]

\[
\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

(4)

Thus, for an infinitesimal rotation,

\[
\psi' \simeq \psi - i\alpha \frac{1}{2} \mathbf{n} \cdot \mathbf{\sigma} \psi,
\]
which means that \( \mathbf{n} \cdot \mathbf{S} = (1/2)\hbar \mathbf{n} \cdot \mathbf{\sigma} \) is the operator corresponding to the component of the spin angular momentum operator along \( \mathbf{n} \).

The Pauli matrices satisfy
\[
\sigma_i \sigma_j = \delta_{ij} I + i \varepsilon_{ijk} \sigma_k, \tag{5}
\]
where \( \varepsilon_{ijk} \) is totally antisymmetric with \( \varepsilon_{123} = 1, i, j, \ldots = 1, 2, 3, \) and there is summation over repeated indices. Since the entries of the Pauli matrices are constant, making use of Eq. (5),
\[
(\sigma \cdot \nabla)^2 = \sigma_i \frac{\partial}{\partial x_i} \sigma_j \frac{\partial}{\partial x_j} = \sigma_i \sigma_j \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} = \begin{pmatrix} \nabla^2 & 0 \\ 0 & \nabla^2 \end{pmatrix},
\]
where the \( x_i \) are Cartesian coordinates; therefore, the equation
\[
-\frac{\hbar^2}{2M} (\sigma \cdot \nabla)^2 \psi + V(r) \psi = i\hbar \frac{\partial \psi}{\partial t} \tag{6}
\]
for the two-component spinor (2) implies that each component of \( \psi (\psi^1 \text{ and } \psi^2) \) satisfies the Schrödinger equation (1), and conversely. However, when there is a magnetic field present this equivalence disappears and Eq. (6) leads to a coupling between the two components of the spinor \( \psi \).

The standard procedure to take into account the interaction of a particle of electric charge \( q \) with an electromagnetic field consists in replacing the partial derivatives \( \partial / \partial x_i \) and \( \partial / \partial t \) by \( \partial / \partial x_i - iqA_i / (\hbar c) \) and \( \partial / \partial t + iq\phi / \hbar \), respectively, where \( A = (A_1, A_2, A_3) \), and \( \phi \) are potentials of the electromagnetic field. In this manner, for a spin-0 charged particle, from Eq. (1) one obtains (see, e.g., Ref. 10)
\[
-\frac{\hbar^2}{2M} \left[ \nabla^2 \psi - \frac{2iq}{\hbar c} \mathbf{A} \cdot \nabla \psi - \frac{iq}{\hbar c} (\nabla \cdot \mathbf{A}) \psi \right] + V(r) \psi + q\phi \psi = i\hbar \frac{\partial \psi}{\partial t} \tag{7}
\]
and, similarly, making use of Eqs. (5), Eq. (6) yields
\[
-\frac{\hbar^2}{2M} \left[ \nabla^2 \psi - \frac{2iq}{\hbar c} \mathbf{A} \cdot \nabla \psi - \frac{iq}{\hbar c} (\nabla \cdot \mathbf{A}) \psi \right] - \left( \frac{q}{\hbar c} \right)^2 \mathbf{A}^2 \psi + V(r) \psi + q\phi \psi = i\hbar \frac{\partial \psi}{\partial t}. \tag{8}
\]
When \( \mathbf{B} = 0 \), Eq. (8) reduces to two independent equations of the form (7), one for each component of the spinor \( \psi \). However, when \( \mathbf{B} \neq 0 \), the components of \( \psi \) are coupled through the term
\[
-\mathbf{B} \cdot \frac{1}{Mc^2} \frac{\hbar}{2} \mathbf{\sigma} \psi.
\]
Recalling that the energy of a magnetic dipole moment \( \mu \) in a magnetic field \( \mathbf{B} \) is equal to \( -\mathbf{\mu} \cdot \mathbf{B} \), it follows that a charged spin-1/2 particle obeying Eq. (8) behaves as if it had a magnetic dipole moment represented by the operator
\[
\mu = \frac{q}{Mc^2} \frac{\hbar}{2} \mathbf{\sigma} = \frac{q}{Mc} \mathbf{S}. \tag{9}
\]
(By contrast with the electric charge, which is a “c-number”, the magnetic dipole moment associated with the particle is an operator.) Equation (9) shows that the ratio of the magnetic dipole moment to the intrinsic angular momentum is equal to
\[
\frac{q}{Mc}. \tag{10}
\]

Before considering an analog of Eq. (6) applicable to spin-3/2 particles, it will be convenient to write Eq. (6) making use of the two-component spinor notation that will be employed in the treatment of spin-3/2 particles (see also Refs. 8 and 9).

The entries of the Pauli matrices (4) will be denoted by \( \sigma_i^A, \sigma_i^B \) \( (A,B,... = 1, 2) \), so that \( \sigma_i^A \) stands for the entry in the \( A \)-th row and \( B \)-th column of the matrix \( \sigma_i \). The spinor indices, such as those of the spinor (2), and of the Pauli matrices, will be lowered or raised following the convention
\[
\phi_A = \varepsilon_{AB} \phi_B, \quad \phi^A = \phi_B \varepsilon^{BA}, \tag{11}
\]
where
\[
(\varepsilon_{AB}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \equiv (\varepsilon^{AB}). \tag{12}
\]
(Thus, \( \phi_1 = \phi^2, \phi_2 = -\phi^1 \).) Hence, \( \varepsilon^A_B = \delta^A_B \) and
\[
\phi_A \phi^A = \varepsilon_{AB} \phi_B \phi^B = -\phi^B \varepsilon^{BA} \phi_A = -\phi^A \psi_B
\]
\[
= -\phi^A \psi_A. \tag{13}
\]

Any tensor with Cartesian components \( t_{ij...k} \) has a spinor equivalent defined by
\[
t_{ABCD...MN} = \frac{1}{\sqrt{2}} \sigma_{iAB} \frac{1}{\sqrt{2}} \sigma^{jCD} \cdots \times \frac{1}{\sqrt{2}} \sigma^k_{MN} t_{ij...k}, \tag{14}
\]
where, following the conventions stated above, \( \sigma_{iAB} = \varepsilon_{AC} \sigma_i^{C} B \). (Since we are considering here Cartesian coordinates only, the tensor indices are lowered or raised by means of the metric tensor \( \delta_{ij} \) and its inverse \( \delta^{ij} \); hence, \( \sigma_{iAB} = \sigma^{AB} \).) An explicit computation shows that
\[
\sigma_{iAB} = \sigma_{iBA}. \tag{15}
\]
Furthermore, since the Pauli matrices have a vanishing trace, from Eq. (5) we obtain \( \text{tr} (\sigma_i \sigma_j) = 2\delta_{ij}, \text{ i.e., } \sigma_i^A \sigma_j^B \sigma^B_A = 2\delta_{ij} \) or, equivalently [see Eq. (13)]
\[
\sigma_{iAB} \sigma_{jAB} = -2\delta_{ij}. \tag{16}
\]
Hence, from Eqs. (14) and (16) we find that, if \( t_{AB} \) and \( s_{AB} \) are the spinor equivalents of \( t_i \) and \( s_i \), respectively
\[
t_{AB} s_{AB} = -t_i s_i. \tag{17}
\]
According to the definition (14), we shall write
\[ \partial_{AB} = \frac{1}{\sqrt{2}} \sigma^A_{\mathbf{C}} \partial_i, \]
(18)
where \( \partial_i \equiv \partial/\partial x^i \). Thus, the Schrödinger–Pauli equation (6) can be expressed as
\[ -\frac{\hbar^2}{2M} \sigma^A_{\mathbf{C}} \partial^B_{\mathbf{C}} \psi + V(\mathbf{r}) \psi = \frac{i\hbar}{\sqrt{2}} \partial_t \psi^A. \]
(19)
We can see that Eq. (19) is equivalent to two decoupled Schrödinger equations using the fact that if \( \phi_{AB} = -\phi_{BA} \), then
\[ \phi_{AB} = \frac{1}{2} \sigma^R_{\mathbf{C}} \epsilon_{AB}. \]
(20)
Indeed, any \( 2 \times 2 \) antisymmetric matrix must be proportional to \( (\epsilon_{AB}) \) [see Eq. (12)] and, as can be readily seen,
\[ \phi_{AB} = \phi_{12} \epsilon_{AB} = \frac{1}{2} (\phi_{12} - \phi_{21}) \epsilon_{AB} = \frac{1}{2} (\sigma^R_{\mathbf{C}} \epsilon_{AB} \epsilon_{AB}, \]
Owing to Eq. (13), and the fact that \( \phi_{AB} = \phi_{BA} \) [see Eqs. (15) and (18)],
\[ \partial_{AC} \partial^C_{\mathbf{B}} = -\partial_{A}^{\mathbf{C}} \partial_{CB} = -\partial_{CB} \partial_{A}^{\mathbf{C}} = -\partial_{BC} \partial_{A}^{\mathbf{C}}, \]
\[ \partial_{AC} \partial^C_{\mathbf{B}} = \frac{1}{2} \epsilon_{AB} \partial^R_{\mathbf{C}} \partial^C_{\mathbf{R}} = -\frac{1}{2} \epsilon_{AB} \partial^R_{\mathbf{C}} \partial_{BC} = \frac{1}{2} \epsilon_{AB} \nabla^2, \]
which is equivalent to
\[ \partial^A_{\mathbf{C}} \partial^C_{\mathbf{B}} = \frac{1}{2} \epsilon^A_{\mathbf{B}}, \]
(21)
so that, in effect, Eq. (19) amounts to
\[ -\frac{\hbar^2}{2M} \nabla^2 \psi^A + V(\mathbf{r}) \psi^A = \frac{i\hbar}{\sqrt{2}} \partial_t \psi^A. \]
When there is an electromagnetic field present, we replace \( \partial^A_{\mathbf{B}} \) by \( \partial^A_{\mathbf{B}} - (iq/h) \mathbf{A}^A_{\mathbf{B}} \) and \( \partial_t \) by \( \partial_t + (iq/h) \phi \) in Eq. (19), and we obtain
\[ -\frac{\hbar^2}{M} \left( \partial^A_{\mathbf{C}} - \frac{iq}{\hbar c} \mathbf{A}^A_{\mathbf{B}} \right) \left( \partial^C_{\mathbf{B}} - \frac{iq}{\hbar c} \mathbf{A}^C_{\mathbf{B}} \right) \psi^B + V(\mathbf{r}) \psi^A + q \phi \psi^A = \frac{i\hbar}{\sqrt{2}} \partial_t \psi^A, \]
which is equivalent to
\[ -\frac{\hbar^2}{M} \left( \partial^A_{\mathbf{C}} \partial^C_{\mathbf{B}} \psi^B - \frac{iq}{\hbar c} (\partial^A_{\mathbf{C}} \mathbf{A}^C_{\mathbf{B}}) \psi^B \right) \]
\[ -\frac{iq}{\hbar c} \mathbf{A}^B_{\mathbf{D}} \partial^A_{\mathbf{C}} \psi^D + \frac{iq}{\hbar c} \mathbf{A}^C_{\mathbf{B}} \partial^A_{\mathbf{C}} \psi^B \]
\[ = \left( \frac{q}{\hbar c} \right)^2 \mathbf{A}^A_{\mathbf{C}} \mathbf{A}^C_{\mathbf{B}} \psi^B + V(\mathbf{r}) \psi^A + q \phi \psi^A = \frac{i\hbar}{\sqrt{2}} \partial_t \psi^A. \]
(22)
In order to reduce this last expression we begin by noticing that [see Eq. (20)]
\[ \partial_{AC} \mathbf{A}^C_{\mathbf{B}} = \frac{1}{2} (\partial_{AC} \mathbf{A}^C_{\mathbf{B}} + \partial_{BC} \mathbf{A}^C_{\mathbf{A}}) \]
\[ + \frac{1}{2} (\partial_{AC} \mathbf{A}^C_{\mathbf{B}} - \partial_{BC} \mathbf{A}^C_{\mathbf{A}}) = \partial_{[AC]} \mathbf{A}^C_{\mathbf{B}} + \epsilon_{AB} \partial^R_{\mathbf{C}} \mathbf{A}^C_{\mathbf{R}}, \]
where the parenthesis denotes symmetrization on the indices enclosed (e.g., \( M_{AB} = (1/2)(M_{AB} + M_{BA}) \)), and the indices between bars are excluded from the symmetrization. The first term in the right-hand side of the last equality is the spinor equivalent of \( (i/\sqrt{2})\nabla \times \mathbf{A} \), which follows from the fact that the spinor equivalent of the Levi-Civita symbol \( \epsilon_{ijkl} \)
is \( \epsilon_{ABCDEG} = (i/\sqrt{2})(\epsilon_{AC} \epsilon_{BE} \epsilon_{DG} + \epsilon_{BD} \epsilon_{CE} \epsilon_{AG}) \) [8,9], while the last term is equal to \( (1/2)\epsilon_{AB} \nabla \cdot \mathbf{A} \) [see Eqs. (13) and (17)]. Making use of Eqs. (13) and (20) we find that
\[ \mathbf{A}^C_{\mathbf{B}} \partial_{AC} + A_{AC} \mathbf{A}^C_{\mathbf{B}} = \mathbf{A}^C_{\mathbf{B}} \partial_{AC} - \mathbf{A}^C_{\mathbf{B}} \partial_{BC} = \epsilon_{AB} \mathbf{A}^C_{\mathbf{R}} \partial_{RC} = \epsilon_{AB} \mathbf{A} \cdot \nabla. \]
Finally, by analogy with Eq. (21), \( \mathbf{A}^C_{\mathbf{B}} \mathbf{A}^C_{\mathbf{B}} = (1/2) \delta^C_{\mathbf{B}} \mathbf{A}^2 \).
Thus, Eq. (22) can be also be written as
\[ -\frac{\hbar^2}{2M} \left( \nabla^2 \psi^A + \frac{\sqrt{2}}{\hbar c} q \mathbf{A}^B_{\mathbf{C}} \psi^B + \frac{iq}{\hbar c} (\nabla \cdot \mathbf{A}) \psi^A \]
\[ - \frac{2iq}{\hbar c} \mathbf{A} \cdot \nabla \psi^A - \left( \frac{q}{\hbar c} \right)^2 \mathbf{A}^2 \psi^A + V(\mathbf{r}) \psi^A \]
\[ + q \phi \psi^A = \frac{i\hbar}{\sqrt{2}} \partial_t \psi^A, \]
(23)
where \( B_{AB} \) denotes the spinor equivalent of \( B \), and one can verify that this expression coincides with Eq. (8).

3. Spin-3/2 particles

A spin-3/2 particle is described by a totally symmetric three-index spinor field, \( \psi^{ABC} \) [see Eq. (25) below], which under rotations transforms according to
\[ \psi^{ABC} = U^A_{\mathbf{R}} U^B_{\mathbf{S}} U^C_{\mathbf{T}} \psi^{RST}, \]
where \( (U^A_{\mathbf{B}}) \) is the SU(2) matrix appearing in Eq. (3), namely
\[ U^A_{\mathbf{B}} = \cos \frac{1}{2} \alpha \delta^A_{\mathbf{B}} - i \sqrt{2} \sin \frac{1}{2} \alpha n^A_{\mathbf{B}}, \]
(24)
and \( n_{AB} \) is the spinor equivalent of the unit vector \( \mathbf{n} \). Hence, for an infinitesimal rotation,
\[ \psi^{ABC} \sim \left( \delta^A_{\mathbf{R}} - \frac{i\alpha}{\sqrt{2}} n^A_{\mathbf{R}} \right) \left( \delta^B_{\mathbf{S}} - \frac{i\alpha}{\sqrt{2}} n^B_{\mathbf{S}} \right) \]
\[ \times \left( \delta^C_{\mathbf{T}} - \frac{i\alpha}{\sqrt{2}} n^C_{\mathbf{T}} \right) \psi^{RST}, \]
\[ \sim \psi^{ABC} - \frac{3i}{\sqrt{2}} \alpha n^A_{\mathbf{R}} \psi^{BCR}, \]
which implies that the operator \( \mathbf{n} \cdot \mathbf{S} \) given by

\[
(n \cdot S)^{ABC} = \frac{3\hbar}{\sqrt{2}} (A_{R}^{A} \psi^{B,C})_{R}
\]

(25)
corresponds to the component of the spin along \( \mathbf{n} \).

By analogy with the Schrödinger–Pauli equation (19), for a spin-3/2 particle we propose the equation

\[
-\frac{\hbar^{2}}{M} \left( \frac{\partial}{\partial t} + V(\mathbf{r}) \right) \psi^{ABC} = i\hbar \partial_{\mathbf{t}} \psi^{ABC} - \frac{q\phi}{\hbar c} \psi^{ABC} = 0,
\]

(26)

which, owing to Eq. (21), means that, in the absence of an electromagnetic field, each component of \( \psi^{ABC} \) satisfies the Schrödinger equation (1). It should be noticed that, even in the absence of an electromagnetic field, the components of the spinor field \( \psi^{ABC} \) may be coupled among themselves if a non-Cartesian basis is employed, in which case the partial derivatives appearing in Eq. (26) have to be replaced by covariant derivatives \([8, 9]\). Instead of the case the partial derivatives appearing in Eq. (26) have to themselves if a non-Cartesian basis is employed, in which case the partial derivatives appearing in Eq. (26) have to be replaced by covariant derivatives \([8, 9]\). Instead of the

By combining Eq. (26) and its complex conjugate one obtains the continuity equation

\[
\frac{\partial}{\partial t} \left( \bar{\psi}^{ABC} \psi^{ABC} \right) + \frac{\hbar}{\sqrt{2}} \partial_{R} S_{BC} A^{AR} \bar{\psi}^{ABC} + \psi^{SBC} A^{AR} \partial_{R} \psi^{ABC} = 0,
\]

(27)

where \( \bar{\psi}^{ABC} \equiv \bar{\psi}^{ABC} \) \([9]\); hence, \( \bar{\psi}^{ABC} \psi^{ABC} \) is real and positive.

As in the case of the relativistic description of spin-3/2 particles, instead of three-index spinors, one can employ wave functions with one spinor index and one tensor index \([4]\). In the present case, we can define

\[
\psi_{A} \equiv -\frac{1}{\sqrt{2}} \sigma^{BC} \psi^{ABC},
\]

then the symmetry of \( \psi^{ABC} \) in its three indices is expressed by the condition \( \sigma^{ABC} \psi_{A} = 0 \).

When there is an electromagnetic field present, we replace \( \delta^{A}_{B} \) by \( \delta^{A}_{B} - \frac{iq}{\hbar c} A_{B}^{A} \) and \( \partial/\partial t \) by \( \partial/\partial t + (iq/\hbar) \phi \), in Eq. (26), which yields

\[
-\frac{\hbar^{2}}{M} \left( \frac{\partial}{\partial t} + \frac{iq}{\hbar c} A_{B}^{A} \right) \left( \chi^{R}_{S} - \frac{iq}{\hbar c} A_{S}^{R} \right) \psi^{S[BC]} + V(\mathbf{r}) \psi^{ABC} + q\phi \psi^{ABC} = \frac{i\hbar}{\partial t} \psi^{ABC}.
\]

(28)

Following the same steps as in Eq. (22) we obtain

\[
-\frac{\hbar^{2}}{2M} \left( \nabla^{2} \psi^{ABC} + \frac{2q}{\hbar c} B^{(A} \psi^{B,C)} \right) + \frac{q}{\hbar c} \psi^{ABC} - \frac{q^{2}}{\hbar^{2}} A^{2} \psi^{ABC} + V(\mathbf{r}) \psi^{ABC} + q\phi \psi^{ABC} = \frac{i\hbar}{\partial t} \psi^{ABC}.
\]

Thus, taking into account Eq. (25), one concludes that in the present case there is an interaction with the magnetic field of the form \( -\mathbf{\mu} \cdot \mathbf{B} \), with

\[
\mathbf{\mu} = \frac{q}{3M_c} \mathbf{S}
\]

(cf. Eq. (9)) that corresponds to the gyromagnetic ratio

\[
\frac{q}{3M_c}.
\]

(30)

Owing to the difference between the gyromagnetic ratios (10) and (30), in both cases, the greatest eigenvalue of the operator \( \mathbf{\mu} \) is given by

\[
\mu_{\text{max}} = \frac{|q| \hbar}{2M}.
\]

(31)

Equations (19) and (26) can be readily generalized for any value of the spin. A spin-\( s \) particle would be represented by a totally symmetric \( 2s \)-index spinor field, \( \psi^{AB...L} \), satisfying

\[
-\frac{\hbar^{2}}{M} \left( \frac{\partial}{\partial t} + \frac{iq}{\hbar c} A_{B}^{A} \right) \left( \chi^{R}_{S} - \frac{iq}{\hbar c} A_{S}^{R} \right) \psi^{S[BC]} + V(\mathbf{r}) \psi^{ABC} + q\phi \psi^{ABC} = \frac{i\hbar}{\partial t} \psi^{ABC}.
\]

(32)

and if the particle has electric charge \( q \), the gyromagnetic ratio would be given by

\[
\frac{q}{2sM_c}.
\]

As pointed out above, under the rotation corresponding to the SU(2) matrix \( (U_{A}^{A}) \), the Cartesian components of a spinor \( \psi^{AB...L} \) transform according to

\[
\psi^{AB...L} = U_{A}^{A} p_{U_{B}^{B}} \cdots U_{L}^{L} R_{PQ...R} \psi^{PQ...R}.
\]

Since each term in Eq. (32) transforms in the same manner as \( \psi^{AB...L} \), the validity of Eq. (32) in a given Cartesian frame implies its validity in any Cartesian frame obtained from the original one by means of a rotation. This can be seen as a consequence of the fact that a contraction of the form \( \delta^{A}_{R} \psi^{RB...L} \) transforms as a 2s-index spinor since

\[
U_{RM} U_{R}^{N} = \delta(U_{A}^{A}) \epsilon_{NM} = \epsilon_{NM}.
\]

[see Eq. (20)] and therefore
\[ \partial_A R \psi^{RL} = (U^A U_{RM} \partial^PM) U^R N U^B Q \cdots U^L S \psi^{NQ} ... S \]
\[ = U_{RM} U^R N U^A P U^B Q \cdots U^L S \partial^PM \psi^{NQ} ... S \]
\[ = \varepsilon_{NM} U^A P U^B Q \cdots U^L S \partial^P N \psi^{NQ} ... S. \]

On the other hand, under the Galilean transformation
\[ r' = r - vt, \quad t' = t, \]
where \( v \) is constant, using the chain rule, one finds that
\[ \frac{\partial}{\partial x_i'} = \frac{\partial}{\partial x_i}, \quad \frac{\partial}{\partial t'} = v_i \frac{\partial}{\partial x_i} + \frac{\partial}{\partial t} \]
then, assuming that
\[ \psi^{AB \cdots L} = \psi^{AB \cdots L} \exp \left( -i \frac{m v \cdot r}{\hbar} + i \frac{m v^2 t}{2\hbar} \right), \]
a straightforward computation, making use of Eq. (21), shows that Eq. (32) is form-invariant under Galilean transformations.

4. Concluding remarks

The preceding equations show that a genuine spin-3/2 charged particle would behave in a different way than an assembly of three charged spin-1/2 particles in a spin-3/2 state, since in the latter case one would have a gyromagnetic ratio equal to that of a single spin-1/2 particle, which differs from (30).

The origin and physical significance of the coincidence of the gyromagnetic ratios of a spin-1/2 charged particle, and of a rotating charged black hole, mentioned in the Introduction are not evident; and the fact that the gyromagnetic ratio derived from Eq. (32) depends on the spin of the particle suggests that this coincidence is not a straightforward consequence of some basic principle.