Order/disorder in brain electrical activity

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Abstract

The processing of information by the brain is reflected in dynamical changes of the electrical activity in time, frequency, and space. Therefore, the concomitant studies require methods capable of describing the quantitative variation of the signal in both time and frequency. Here we present a quantitative EEG (qEEG) analysis, based on the Orthogonal Discrete Wavelet Transform (ODWT), of generalized epileptic tonic-clonic EEG signals. Two quantifiers: the Relative Wavelet Energy (RWE) and the Normalized Total Wavelet Entropy (NTWS) have been used. The RWE gives information about the relative energy associated with the different frequency bands present in the EEG and their corresponding degree of importance. The NTWS is a measure of the order/disorder degree in the EEG signal. These two quantifiers were computing in EEG signals as provided by scalp electrodes of epileptic patients. We showed that the epileptic recruitment rhythm observed for generalized epileptic tonic-clonic seizures is accurately described by the RWE quantifier. In addition, a significant decrease in the NTWS was observed in the recruitment epoch, indicating a more rhythmic and ordered behavior in the brain electrical activity.

Keywords: EEG; epileptic seizures; time-frequency signal analysis; wavelet analysis; signal entropy.

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1. Introduction

Human brain electrical activity can be measured from the scalp, in a non-invasive way, by means of electroencephalography (EEG). Due to the conductive properties of biological tissue, this activity reflects the effect of proximal and distal sources of synchronized neuronal activities [1]. Recently, oscillatory EEG activity has been discussed in relation with functional neuronal mechanisms. In this regard, it is of major interest to investigate how brain electric oscillations get synchronized in pathological or physiological brain states (e.g., epileptic seizures, sleep-wake stages, etc.), or by external and internal stimulation (event related potentials (ERP) or evoked potentials (EP)). This issue can be addressed by applying methods of system’s analysis to the EEG signals, because changes in EEG activity occur in temporal relation to triggering events, and could be thought of as transitions from disordered to ordered states (or vice versa).

A natural approach to quantify the degree of order of a complex signal is to consider its spectral entropy, as defined from the Fourier power spectrum [2]. The spectral entropy is a measure of how concentrated or widespread the Fourier power spectrum of a signal is. An ordered activity, like a sinusoidal signal, is manifested as a narrow peak in the frequency domain. This concentration of the frequency spectrum in one single peak corresponds to a low entropy value. On the other extreme, a disordered activity (e.g., the one generated by pure noise or by a deterministic chaotic system) will have a wide band response in the frequency domain that is reflected in higher entropies. However, the Fourier transform (FT) requires stationarity of the signal as well, and EEGs are highly non stationary. Furthermore, the FT does not yield the time evolution of the pertinent frequency patterns. Consequently, the spectral entropy does not get defined as a function of time.

The disadvantages of the spectral entropy defined from the FT can be partially overcome by using a short-time Fourier transform (STFT). Powell and Percival [2] defined a time evolving entropy from the STFT by using a Hann-window. With this approach, the FT is applied to time-evolving windows of a few seconds of data refined with an appropriate function, so that the time-evolution of the frequencies can be followed. The stationarity require-
ment is partially satisfied by considering the signals as quasi-stationary for a few seconds. Due to the uncertainty principle [3,4], one critical limitation appears when windowing data: if the window is too narrow, the frequency resolution will be poor. Conversely, if the window is too wide, the time localization will be less precise.

All the above mentioned difficulties can be overcome by appeal to the wavelet transform [3,4], an efficient time-frequency decomposition method. In particular, the orthogonal discrete wavelet transform (ODWT) makes no assumptions about a record’s stationarity. The only input needed is the time series itself. If the entropy is computed via the wavelet transform, the time evolution of frequency patterns can be followed with an optimal time-frequency resolution [5,6]. The ensuing entropy-form, based on the wavelet transform, is called the “wavelet entropy”. It reflects the degree of order/disorder of the signal. The wavelet entropy appears thus as a natural measure of order for EEG signals, more specifically of the synchrony of the group of cells involved in the different neural responses [5,6].

2. Clinical data and experimental setup

A scalp EEG signal is essentially a nonstationary time series that presents artifacts due to electrooculogram (EOG), electromyogram (EMG) and electrocardiogram (ECG), among others [1]. Artifacts related to muscle contractions are specially troublesome in the case of tonic-clonic epileptic seizures, where they reach very high amplitudes that contaminate the whole seizure recording. Sometimes artifacts are presented during a few seconds and can be obviated because they obscure only a small portion of the EEG. In other cases, almost the total signal appears obscured by them, and very little information about the underlying brain activity can be extracted. An example of this kind of scalp EEG signals are the ones corresponding to an epileptic tonic-clonic seizures [1].

A tonic-clonic (TC) seizure is characterized by violent muscle contractions. Initial massive tonic spasms are replaced seconds later by the clonic phase with violent flexor movements and characteristic rhythmic spasms towards the ending of the seizure. In these seizures, artifacts related to muscle contractions are especially troublesome because they reach very high amplitudes [1]. In fact, not only do they limit the traditional visual analysis to the pre- and post-ictal periods, but they also restrict the application of some mathematical methods.

Analysis of the brain activity during this seizure has been previously performed only in special circumstances, such as in patients treated with curare (an inhibitor of the muscle responses) [7,8] or by eliminating the high frequency muscle activity with the use of traditional filters [9]. Gastaut and Broughton [7] described a frequency pattern during a tonic-clonic epileptic seizure from patients with muscle relaxation from curarization and artificial respiration. After a short period (which may be as short as 1 – 3 s) characterized by phase desynchronization they found an “epileptic recruiting rhythm” at about 10 Hz [8] with a rapidly increasing amplitude dominating the EEG; later, as the seizure ends, there is a progressive increase of the lower frequencies associated with the clonic phase. About 10 s after the seizure onset, lower frequencies delta and theta (0.5 – 3.5 Hz) are observed that gradually diminish their activity. The clonic activity corresponds with generalized polyspike bursts at each myoclonic jerk. Very slow irregular delta activity dominates then the EEG, accompanied with a gradual frequency increase of the theta (3.5 – 7.5 Hz) and alpha bands (7.5 – 12.5 Hz), indicative of the end of the seizure.

Twenty tonic-clonic epileptic seizures from 8 epileptic patients admitted for video-EEG monitoring were analyzed. The subjects consisted of 4 males and 4 females, aged 30.87 ± 15.27 (mean ± SD; range 6–51) with a diagnosis of pharmaco-resistant epilepsy and no other accompanying disorders. Scalp bimastoideal reference were applied following the 10-20 international system. Each signal was digitized at 409.6 Hz through a 12 bit A/D converter and filtered with an “antialiasing” 8 pole low pass Bessel filter, with a cutoff frequency of 50 Hz. Then, the signal was digitally filtered with a 1–50 Hz bandwidth filter and stored, after decimation, at \( \omega_s = 102.4 \) Hz (sample frequency) in a PC hard drive.

Recordings were done under video control to have an accurate determination of the different stages of the seizure. The different stages of EEG signals were determined by the physician team. Off-line analysis was performed with characterization of semiological features, timing of the onset and definition, when possible, of the anatomical focus for each event. Analysis for each event included 60 s of EEG before the seizure onset and 120 s of ictal and post-ictal phases. All 180 s were analyzed at the C4 derivation, this electrode chosen after visual inspection of the EEG (by the physician team) as the one with the least number of artifacts. In all the cases, the time intervals with artifact for pre-ictal stage were marked by the physician team.

As an example, in Fig. 1 we present a scalp EEG signal corresponding to a tonic-clonic epileptic seizure recorded over the right central region (C4 channel). In this record, pre-ictal phase is characterized by a signal of 50 \( \mu \)V. The seizure starts at 74 s, with a discharge of slow waves superimposed with low voltage fast activity. This discharge lasted approximately 11 s and with a mean amplitude of 100 \( \mu \)V. Afterwards the seizure spreads, making the analysis of the EEG more complicated due to muscle artifacts. It is possible, however to establish the beginning of the clonic phase at approximately 120 s and the end of the seizure at 158 s where there is an abrupt decay of the signal amplitude.

3. Wavelet analysis

Wavelet analysis is a method which relies on the introduction of an appropriate basis and a characterization of the signal by the distribution of amplitude in this basis. If the basis is required to be a proper orthogonal basis, any arbitrary function
The correlated decimated discrete wavelet transform provides a nonredundant representation of the signal and its values constitute the coefficients in a wavelet series. These wavelet coefficients provide full information on the signal in a simple way and a direct estimation of local energies at different scales. Moreover, the information can be organized in a hierarchical scheme of nested subspaces called multiresolution analysis. In the present work we employ orthogonal cubic spline functions as mother wavelet, \( \psi \). Among several alternatives, cubic spline functions are symmetric and combine in a suitable proportion smoothness with numerical advantages.

In the following the signal is assumed to be given by the sampled values \( S = \{ s_n(n), n = 1, \ldots, M \} \), corresponding to a uniform time grid with sampling time (frequency) \( t_s(\omega_s) \). If the decomposition is carried out over all resolution levels \( N = \log_2(M) \), the wavelet expansion will read

\[
S(t) = -\frac{1}{\sum_{j=-N}^{1} N} \sum_{k} C_j(k) \psi_{j,k}(t) = -\frac{1}{\sum_{j=-N}^{1} N} r_j(t),
\]  

where the wavelet coefficients \( C_j(k) \) can be interpreted as the local residual errors between successive signal approximations at scales \( j \) and \( j+1 \), and \( r_j(t) \) is the residual signal at scale \( j \). It contains the information of the signal \( S(t) \) corresponding to the frequencies \( 2^j-1 \omega_s \leq |\omega| \leq 2^j \omega_s \).

Since the family \( \{ \psi_{j,k}(t) \} \) is an orthonormal basis for \( L^2(\mathbb{R}) \), the concept of energy is linked with the usual notions derived from Fourier’s theory. The wavelet coefficients are given by \( C_j(k) = \langle S, \psi_{j,k} \rangle \) and the energy, at each resolution level \( j = -1, \ldots, -N \), will be the energy of the detail signal

\[
E_j = \| r_j \|^2 = \sum_{k} |C_j(k)|^2.
\]  

The total energy can be obtained in the fashion

\[
E_{\text{tot}} = \| S \|^2 = -\sum_{j=-N}^{1} \sum_{k} |C_j(k)|^2 = -\sum_{j=-N}^{1} E_j.
\]  

Finally, we define the normalized \( p_j \)-values, which represent the relative wavelet energy

\[
p_j = E_j / E_{\text{tot}}
\]  

for the resolution levels \( j = -1, -2, \ldots, -N \). The \( p_j \)'s yield, at different scales, the probability distribution for the energy. Clearly,

\[
\sum_{j=-N}^{1} p_j = 1
\]  

and the distribution \( \{ p_j \} \) can be considered as a time-scale density that constitutes a suitable tool for detecting and characterizing specific phenomena in both the time and the frequency planes.

The Shannon entropy [10] gives a useful criterion for analyzing and comparing probability distribution. It provides a measure of the information contained in any distribution. We define the Normalized Total Wavelet Entropy (NTWS) [5, 6] as

\[
S_{\text{WT}} = -\sum_{j=-N}^{1} p_j \ln|p_j| / S_{\text{max}},
\]  

with \( S_{\text{max}} = \ln[N] \) the normalization constant. The NTWS appears as a measure of the degree of order/disorder of the signal, so it can provide useful information about the underlying dynamical process associated with the signal. Indeed, a very ordered process can be represented by a periodic monofrequency signal (signal with a narrow band spectrum). A wavelet representation of such a signal will be resolved at one unique wavelet resolution level, i.e., all relative wavelet energies will be (almost) zero except at the wavelet resolution level which includes the representative signal frequency. For this special level the relative wavelet energy will (in our chosen energy units) almost equal unity. As a consequence, the NTWS will acquire a very small, vanishing value. A signal  

\[\text{FIGURE 1. Scalp EEG signal for an epileptic tonic-clonic seizure, recorded at central right location (C4). The seizure starts at 74 s and the clonic phase at 117 s. The seizure ends at 158 s.}\]
generated by a random process can be taken as representative of a very disordered behavior. This kind of signal will have a wavelet representation with significant contributions coming from all frequency bands. Moreover, one could expect that all contributions will be of the same level. Consequently, the relative wavelet energy will be almost equal at all resolutions levels, and the NTWS will acquire its maximum possible value.

In order to follow the temporal evolution of the above defined quantifiers (RWE and NTWS) the analyzed signal is divided into non-overlapping temporal windows of length $L$ and, for each interval $i (i = 1, \cdots, N_T$, with $N_T = M/L$). Appropriate signal-values are assigned to the central point of the time window. In the case of a diadic wavelet decomposition, the number of wavelet coefficients at resolution level $j$ is two times smaller than at the previous, $j + 1$, one. The minimum length of the temporal window will therefore include at least one wavelet coefficient in each scale.

The wavelet energy at resolution level $j$ for the time window $i$ is given by

$$E_j^{(i)} = \sum_{k=(i-1)L+1}^{iL} |C_j(k)|^2 \quad \text{with} \quad i = 1, \cdots, N_T,$$

while the total energy in this time window will be

$$E_{tot}^{(i)} = \sum_{j=N}^{-1} E_j^{(i)}.$$

The time evolution of the relative wavelet energy (RWE) and the normalized total wavelet entropy (NTWS) will given by

$$p_j^{(i)} = E_j^{(i)}/E_{tot}^{(i)},$$

$$S_{WT}^{(i)} = -\sum_{j=-N}^{-1} p_j^{(i)} \cdot \ln[p_j^{(i)}]/S_{max}.$$

4. Results and discussion

Wavelet analysis is a suitable tool for detecting and characterizing specific phenomena in time and frequency planes. Then, neuroelectric activity (EEG time series) was transformed to the time-frequency domain by means of the orthogonal discrete wavelet transform (ODWT) [5]. EEG spectral analysis is traditionally performed by studying different frequency bands with well defined boundaries. Some small variations can be found, according to the particular experiment under consideration. Absolute and relative intensities of these bands are usually analyzed and correlated with different pathologies. In this work we define six frequency bands for an appropriate wavelet analysis within the multiresolution scheme to be used. We denoted these band-resolution levels by $B_j (|j| = 1, \cdots, 6)$. Their frequency limits, time resolution, as well as their correspondence with traditional EEG frequency bands, are given in Table I. Note that the coefficients were non-overlapping for each scale or frequency band.

Electrical muscular activity can be associated with frequencies in the range frequency bands $B_1$ and $B_2$, at wavelet resolution levels $j = -1$ and $-2$, respectively [1]. Then the contributions corresponding to $B_1$ and $B_2$, containing high frequency artifacts related to muscular activity that blurred the EEG, were not taken into account for the evaluation of the wavelet based quantifiers. Although high frequency brain activity is thereby also eliminated, its contributions during the ictal stage is not as strong as it is for middle and low frequencies. This has been conclusively demonstrated in [11, 12]. In order to make a behavior “quantification” we divided the total signal in time-window intervals. In the present study the time-window width employed was of 256 data $= 2.5$ s.

Figure 2 displays the RWE without electromyographic contributions (bands $B_3$ to $B_6$ are displayed). We see that the pre-ictal phase is characterized by a dominance of low rhythms $B_5$ and $B_6$. The seizure starts at 74 s with a discharge of slow waves superimposed to low voltage fast activity. This discharge lasts approximately 11 s and produces
a marked “activity-rise” in the frequency bands $B_3$ and $B_6$ (delta band), which reaches 85% of the RWE. Starting at 85 s, the low frequency activity, represented in our analysis by $B_5$ and $B_6$, decreases abruptly to relative values lower than 10% and 5% respectively, while the other frequency bands (theta and alpha bands) become more important. We also observe in Fig. 2 that the start of the clonic phase is correlated with increased activity in the $B_4$ and $B_5$ frequency bands. After 140 s, when clonic discharges become less frequent, the $B_3$ activity rises up again till the end of the seizure, when the $B_6$ frequency activity also increases in very rapid fashion and both frequency bands become clearly dominant. The $B_3$ and $B_6$ (delta band) frequency bands maintain this predominance throughout the post-ictal phase.

We conclude from this example that the seizure was dominated by the middle frequency bands $B_3$ and $B_4$ (alpha and theta rhythms, 12.8 – 3.2 Hz), with a corresponding abrupt activity decrease in the low frequency bands $B_5$ and $B_6$ (delta rhythm, 3.2 – 0.8 Hz) [11, 12]. Clearly, this behavior can be associated with the putative “epileptic recruiting rhythm” of [7, 8]. One important point to note is that our results were obtained with scalp recordings and without the use of curare or any filtering method. Since intracranial recordings are nearly free of artifacts, the fact that the same pattern [13] was seen in both situations reinforces the idea that the results obtained with scalp electrodes were not a spurious effect of muscle activity.

For the data presented above, the ensuing NTWS, as a function of time, is depicted in Fig. 3. The dotted line represents the time evolution of the NTWS (all frequency bands are included), while the continuous line corresponds to results which ignore contributions due to high frequency bands ($B_1$ and $B_2$). It is interesting to observe the behavior of the NTWS during the first 11 s following the seizure onset. We see that in this time interval the NTWS exhibits increasing values if all wavelet frequency bands are included. Comparison is to be made with NTWS values in the pre-ictal stage.

If the wavelet frequency bands $B_1$ and $B_2$ (bands that mainly reflect muscular activity) are not included, the largest NTWS value is lower than that for the ictal onset. Thus, the behavior of the NTWS following the seizure onset is compatible with an increase in the degree of disorder of the system induced by a high frequency activity. Superimposed low and medium frequency activities, however, are responsible for the “remaining-signal’s” more ordered behavior.

The NTWS behavior after 85 s (in both cases, with and without inclusion of high frequency bands) is indicative of the fact that the system exhibits a tendency to be more “ordered”. This tendency is better appreciated without muscle activity. Moreover, note that the NTWS in the last case adopts a minimum value around 120 s, in coincidence with the beginning of the clonic phase. The peak observed in the NTWS at ~ 140 s could be associated with the disappearance of the epileptic recruitment rhythm. After this point, the NTWS displays increasing values until 158 s, which is defined as the seizure ending time. We see that the NTWS for the post-ictal stage displays almost constant values, comparable to those obtained for the pre-ictal stage.

Similar analysis were performed in the other 19 EEG time series taken in the channel $C_4$, corresponding to 8 different patients. In Figs. 4 and 5 time averages values ($mean \pm SD$)

![Figure 3](image-url)  
**Figure 3.** Normalized total wavelet entropy’s time evolution. The dotted and the solid lines represent, respectively, the NTWS time evolution with and without the contribution of the frequency bands $B_1$ and $B_2$. The vertical lines represent the start and ending of the epileptic seizure.

![Figure 4](image-url)  
**Figure 4.** RWE temporal average values ($mean \pm SD$) over all time windows for $B_1$ and $B_6$ (delta activity) corresponding to pre-ictal and ictal stages, for the 20 tonic-clonic epileptic seizures analyzed. Electrical muscular activity have not taken into account by setting to zero the contribution of frequency bands $B_1$ and $B_2$ in the evaluation of total wavelet energy. For the pre-ictal stage, time intervals that present artifacts have been excluded.
over pre-ictal stage (excluding the intervals with artifacts) and ictal stage corresponding to RWE for frequency bands $B_5$ and $B_6$, and NTWS respectively, are shown. Electrical muscular activity has been not taken into account by setting to zero the contribution of frequency bands $B_1$ and $B_2$ in the evaluation of total wavelet energy. Epileptic recruitment rhythm characterized by significant decrease in the RWE corresponding to frequency band $B_5$ and $B_6$ is observed in all 20 tonic-clonic epileptic seizures analyzed (see Fig. 4). Moreover, one can associate a more robust degree of order to the EEG activity during the ictal than during the pre-ictal stages (see Fig. 4), compatible with a dynamic process of synchronization in the brain activity. This behavior may be thought as induced by an hypothetical epileptic focus which generates the observed epileptic recruitment rhythm.

One critical point is the possible distortion due to spatial propagation of the seizure, since data from the $C_4$ electrode was analyzed and the sources of the seizures were mostly in temporal locations (number of patients and source of seizures: 1 right temporal; 3 left temporal; 2 bitemporal; 2 non localized - see Table I of our previous work [11]). In order to overcome this, quantifiers based on ODWT were also applied to $T_3$ and $T_4$ electrodes, obtaining similar results to the ones reported with $C_4$ electrode. That is even though these electrodes present more artifact, compared with $C_4$, the “recruitment epileptic rhythm” was observed, as well as decreased WS values for the ictal stage compared with pre-ictal one.

5. Conclusions

The present work describes the use of quantitative parameters derived from the orthogonal discrete wavelet transform as applied to the analysis of brain electrical signals. The relative wavelet energy provides information about the relative energy associated with different frequency bands present in the EEG and enables one to ascertain their corresponding degree of importance. The normalized total wavelet entropy carries information about the degree of order/disorder associated with a multi-frequency signal response. In addition, the time evolution of these quantifiers gives information about the dynamics associated with the EEG records.

In particular we have shown that the epileptic recruitment rhythm behavior reported by Gastaut and Broughton [8] for generalized epileptic tonic-clinic seizures is accurately described by the relative wavelet energy concept. Moreover, the present studies do not require the use of curare or of digital filtering. In addition, a significant decrease in the normalized total wavelet entropy was observed in the recruitment epoch, indicating a more rhythmic and ordered behavior of the EEG signal, compatible with a dynamical process of synchronization in the brain activity.

One interesting point to note is that although the grouping in frequency bands implies a loss of frequency resolution, this procedure can be more useful than a study of single frequencies or peaks, due to the relation between frequency bands and functions or sources in the brain. In this context, the relative wavelet energy allows for an easy interpretation of several minutes of frequency variations in a single display, something that is sometimes difficult to achieve with traditional scalp EEGs.

Being independent of the amplitude or the energy of the signal, the wavelet entropy yields new information about EEG signals in comparison with that obtained by using frequency analysis or other standard methods. The normalized total wavelet entropy has the following advantages: (i) In contrast to the spectral entropy, the normalized total wavelet entropy is capable of detecting changes in a non-stationary signal due to the localization characteristics of the wavelet transform; (ii) In comparison with dimensional analysis and Lyapunov exponents (which are only defined for stationary behaviors), or with dimensionality and chaoticity measures (stationary constraints removed), the computational time required for normalized total wavelet entropy studies is significantly shorter. The algorithm for normalized total wavelet entropy evaluation involves just the use of the wavelet transform in a multiresolution framework; (iii) Contaminating noises’ contributions (if they are basically concentrated in some frequency bands) can be easily eliminated; and last but not least, (iv) the normalized total wavelet entropy is parameter-free.

The use of the quantifiers based on time-frequency methods (like ODWT) can contribute to the analysis of brain electrical responses and may also lead to a better understanding of their dynamics. Certainly, the use of these quantifiers is not intended to replace conventional EEG analyzes, but to provide further insights into the underlying brain mechanisms.
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