Using MILP Tools to Study R&D Portfolio Selection Model for Large Instances in Public and Social Sector

Usando Herramientas de MILP para Estudiar el Modelo de Selección de Portafolios R&D para Casos de Grandes Carteras de Proyectos en el Sector Social

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Abstract

In this paper a mixed-integer linear programming (MILP) model is studied for the bi-objective public R&D projects portfolio problem. The proposed approach provides an acceptable compromise between the impact and the number of supported projects. Lagrangian relaxation techniques are considered to get easy computable bounds for the objectives. The experiments show that a solution can be obtained in less than a minute for instances comprising of up to 25,000 project proposals. This brings significant improvement to the previous approaches that efficiently manage instances of a few hundred projects.

Keywords: R&D projects portfolios, mixed integer programming, multi-objective optimization.

Resumen

En este trabajo se presenta un modelo de programación lineal entera mixta (MILP) para el problema del portafolio de proyectos públicos R&D bi-objetivo. El enfoque propuesto provee un punto medio entre el impacto y el número de los proyectos. Se consideran técnicas de relajación Lagrangiana para obtener cotas fácilmente calculables para los valores objetivos. La experimentación muestra que puede obtenerse una solución en menos de un minuto incluso para casos de carteras de más de 25,000 proyectos propuestos. Esto implica una mejora significativa a los enfoques previos que resuelven eficientemente casos con sólo algunos cientos de proyectos.

Palabras clave: Portafolios de proyectos, programación entera mixta, optimización multiobjetivo.

1 Introduction

Portfolio optimization problems are very well known and intensively studied in capital investment, stock market, and private-sector R&D project selection, to mention a few. Somewhat surprisingly, the public sector has not shown a similar interest on this topic; the researches in the public-sector rather approach their project-selection problems by simple heuristics. Typically, public R&D projects can be considered as statistically independent with very small (or zero) correlation. Moreover, the amount of resources sufficient to realize the project is not known exactly and the budget is frequently overestimated by the proponent.

In recent years various models and solutions to the R&D portfolio optimization problem were proposed [Hsu et al., 2003; Ringuest et al., 2004] and corresponding decision support systems were considered [Fernández et al., 2006; Stummer and Heidenberger, 2003; Tian et al., 2005]. But to the best of our knowledge, only the work of Fernández et al. (2004) and Fernández et al. (2006) and that of Navarro (2005) propose methods that have a theoretical foundation and robust heuristics. Those approaches are based on mathematical decision theory, fuzzy logic, rough sets, evolutionary optimization, such different tools incorporated in a decision support system. However, the proposed techniques lack scalability and only work with medium-sized instances (with at most 400 projects). Moreover, the existing approaches are directed towards the portfolio quality as the unique criterion. This can be acceptable only if the decision maker provides reliable and perfectly consistent information on his/her preferences.

In practice, the information of preferences provided by the decision maker is generally rough and partially inconsistent. Litvinchev and López (2008) and Litvinchev et al. (2008) have shown that by introducing the *quantity*

of funded projects as an additional objective in the portfolio-optimization model this issue can be resolved.

When solving a multi-objective portfolio-optimization model, one needs to optimize repeatedly the portfolio according to every objective included. Hence the optimization method has to be highly efficient: even for large instances, the response time should be very short to allow the decision maker to be concentrated and to allow him/her to explore a larger fraction of the set of feasible portfolios.

In this work we consider an effective approach to the multi-objective portfolio optimization for large instances arising in real world public sector (with several thousands of project proposals). For example, thousands projects are considered every year by scientific foundations such as the NSF in the United States and the CONACyT in Mexico. In contrast to the work of Navarro (2005), presenting a genetic algorithm to explore the space of possible portfolios, the proposed method is based on the mixed-integer linear-programming (MILP). This model differs from the one proposed by Litvinchev and López (2008) in the way the integer variables are introduced. Using binary variables we give here an extremal MILP characterization of the original nonlinear discontinuous objective function. Lagrangian relaxations are considered to obtain simple computable bounds for the optimal values. Bi-objective formulation taking into account the quantity of funded projects is also studied using MILP approach.

2 The portfolio-optimization problem in the public sector

The proposed model is based on the normative solution approach proposed by Fernández et al. (2004), where a *nonlinear preference model* is constructed from the *fuzzy generalization* of the classic scheme of 0-1 programming and from the multi-attribute decision theory.

For hundreds of variables, the complexity of the nonlinear optimization problem is *not* manageable by traditional algorithms. It has been addressed with genetic algorithms and neural networks [Navarro, 2005], and more recently by differential evolution [Castro, 2007]. Litvinchev and López (2008) reformulate the problem as a *multi-objective* problem on a *mixed-integer linear-programming model*.

The original non-linear model can be stated, in matrix form, as follows [Fernández et al., 2004; Litvinchev and López, 2008; Litvinchev et al., 2008]:

$$
\begin{aligned}
\max \quad & (\eta, \varphi) \\
& \text{such that} \\
& c^T \cdot \delta \geq \eta \\
& w^T \cdot \mu(\delta) \geq \varphi \\
& \underline{p} \leq q^T \cdot d \leq \overline{p} \\
& h^T \cdot d \leq t \\
& \delta \cdot m \leq d \leq \delta \cdot M
\end{aligned}
\tag{1}
$$

where η and φ represent the objectives: quantity of funded projects and portfolio quality respectively. The total amount of funds available for distributing among projects is t.

The components of the vector δ take binary values, depending on whether a project j is funded ($\delta_j = 1$) or not ($\delta_j = 0$). The decision variables of the vector d are the *funding assignments*: component d_j corresponds to the amount of funding assigned to project j.

The function μ is a *fuzzy predicate* that models the level of funding of a project. For the rest of this paper, we say that the funding of the project is *sufficient* if d_j, being the amount of funds assigned, belongs to a certain interval, $m_j \leq d_j \leq M_j$, where M_j is the amount requested by the proponent and m_j is the minimal funding with which it is possible to carry out project j. See Figure 1 for an illustration of the fuzzy predicate. The components of the vectors m and M are m_j and M_j, respectively.

Each project must be assigned to exactly one *sector* (being an application area, scientific discipline, etc.) The

division into sectors can incorporate organizational structure or the aspiration to balance the portfolio among different disciplines. The *funding constraints* for each sector are represented by the vectors $\underline{p}$ and $\overline{p}$ that contain the minimum and the maximum budget (respectively) to be assigned to each of the sectors.

The *impact measures* of the projects are the components of the weight vector w. Information on how the projects have been evaluated is contained in their impact measures. Thus, the decision variables are d, δ, while c, w, $\underline{p}$, $\overline{p}$, m, M, q, h, and t are known coefficient vectors.
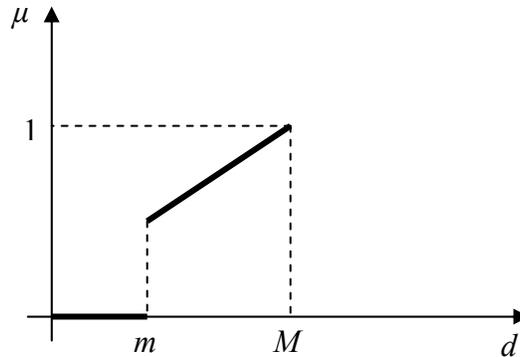

Fig. 1. A fuzzy piecewise linear function to model sufficient funding of a project

The complexity of solving this model lies in the non-linearity and the *discontinuous* nature of both the feasible region and the quality-characterizing objective function. These unfortunate features make the model very hard to optimize with heuristic methods and directly rules out the use of exact methods. However, considering the problem structure, it may be possible to divide the objective function and the feasible region to less complex subproblems [Hooker, 2007; Jain and Grossman, 2001]. In the next section we present a linear mixed-integer representation of the non-linear discontinuous quality objective function and transform the original problem to a linear bi-objective mixed integer problem.

3 The mixed-integer linear model

Consider first the problem (1) taking into account only quality objective. Bearing in mind Figure 1 we will consider the quality of the project as a non-negative function $z(x)$ defined for all $0 \leq x \leq M$, monotonously increasing for $x \in [m, M]$ with $z(M) = 1$, $0 < z(m) < 1$, and $z(x) = 0$ for $0 \leq x < m$. Respectively, expected utility of the project is defined as $wz(x)$, where $w > 0$ is a known constant. The objective is to maximize the sum of individual utilities (portfolio quality) subject to limited funds.

To formalize the problem we first give an extremal presentation of the nonlinear function $z(x) \geq 0$ defined for $x \in [0, M]$ as follows:

$$z(x) = \begin{cases} 0 \text{ for } x < m \\ \alpha + \gamma x \text{ for } m \leq x \leq M \end{cases}. \tag{2}$$

Here $\gamma > 0$, while α is free. In what follows we set $z(m) = 0.5$ Combining this with $z(M) = 1$ yields $\gamma = \frac{1}{2(M-m)}$ and $\alpha = 1 - \frac{M}{2(M-m)}$.

Proposition 1. For any fixed $x \in [0, M]$ the function $z(x)$ defined in (2) coincides with the optimal value of the

following problem:

$$z^*(x) = \max z$$
$$z \leq \alpha y + \gamma x,$$
$$z \leq y, \quad x \geq my,$$
$$0 \leq z, \quad y \in \{0,1\}.$$

(3)

Proof. *a)* Suppose that $0 \leq x < m$. Then by $x \geq my$ we need to set $y = 0$. Hence by $z \leq y$ we have $z \leq 0$, while $z \leq \alpha y + \gamma x$ yields $z \leq \gamma x$ with $\gamma x \geq 0$. Combining with the last constraint in (3), we get $z = 0$.

b) Suppose that $m \leq x \leq M$. By definition of *α*, *γ*, for these values of *x* we have $0.5 \leq \alpha + \gamma x \leq 1$. Constraint $x \geq my$ is fulfilled for any $y \in \{0,1\}$. If $y = 0$, then $z = 0$ as before. For $y = 1$ the problem (3) reduces to $\max\{z \mid z \leq \alpha + \gamma x\}$ giving $z = \alpha + \gamma x > 0$. Hence, for $m \leq x \leq M$, the optimal $y = 1$ and $z^*(x) = \alpha + \gamma x$ as desired.

Suppose we have *J* projects, each characterized by its own vector of parameters $(w, \alpha, \gamma, m, M)_j$, $j = 1, 2, \ldots, J$. Based on Proposition 1, maximizing the overall expected utility subject to linear constraints $Ax \leq b$ on funding $x = (x_1, \ldots, x_j, \ldots, x_J)$ can be stated as

$$F^* = \max \sum_j w_j z_j$$

(4)

$$\left. \begin{array}{l} z_j \leq \alpha_j y_j + \gamma_j x_j, \\ z_j \leq y_j, \quad x_j \geq m_j y_j, \\ 0 \leq z_j, \quad y_j \in \{0,1\}, \\ \quad 0 \leq x_j \leq M_j \end{array} \right\} \quad j = 1, 2, \ldots, J$$

(5)

$$Ax \leq b$$

(6)

By linear constraints (6) we can represent, for example, constraints for the overall funding of the projects. If the projects are grouped in certain areas of specializations, then similarly we can state constraints for the total funding in a particular area.

Typically, the number of applicant projects is very large. Meanwhile, the number of project areas (i.e., the sectors into which the proposals are divided) is relatively small. That is, the mixed-integer problem defined by (4)-(6) has a large number of variables and relatively few linear constraints 6. In real decision-making processes the problem of Equations 4-6 has to be solved repeatedly for different values of original data. So it is very desirable to have a fast method at least to estimate the optimal value F^*.

4 Lagrangian Relaxation and Bounds

Most large scale optimization problems exhibit a structure that can be exploited to construct efficient solution techniques. In one of the most general and common forms of the structure the constraints of the problem can be divided into "easy" and "complicated". In other words, the problem would be an "easy" problem if the complicating constraints could be removed. One typical example is a block-separable problem decomposing into a number of smaller independent subproblems if the binding constraints could be relaxed.

A well-known way to exploit this structure is to form the Lagrangian relaxation with respect to complicating constraints. That is, the complicating constraints are relaxed and a penalty term is added to the objective function to discourage their violation. Typically, the penalty is a linear combination of corresponding slacks with coefficients called Lagrange multipliers. The optimal value of the Lagrangian problem, considered for fixed multipliers, provides

an upper bound (for maximization problem) for the original optimal objective. The problem of finding the best, i.e. bound minimizing Lagrange multipliers, is called the Lagrangian dual. The literature on Lagrangian relaxation is quite extensive. We refer here only to a few survey papers [Fisher, 1985; Rangioni, 2005; Guignard, 2003; Lemaréchal, 2001].

To derive an upper bound for F^* we will use the Lagrangian relaxation, dualizing the constraints (6) with multipliers $u \geq 0$, where the vectors u and b are dimensioned correspondingly. The Lagrangian problem, considered for fixed multipliers, has the form

$$L(u) = u^t b + \max \left\{ \sum_j (w_j z_j - c_j x_j) \right\} \tag{7}$$
$$\text{subject to the constraints (5),}$$

where $c_j \equiv c_j(u)$ is jth component of the vector $A^t u$. We have $F^* \leq L(u)$ for all $u \geq 0$.

This Lagrangian problem decomposes into J independent subproblems in variables $(x, y, z)_j$. The later subproblems have the form

$$\ell = \max wz - cx$$
$$z \leq \alpha y + \gamma x,$$
$$z \leq y, \quad x \geq my, \tag{8}$$
$$0 \leq z, \quad y \in \{0,1\},$$
$$0 \leq x \leq M,$$

where we have omitted the indices j to simplify the notation.

Problem (8) is solved by inspection. If $y = 0$, then $z = 0$, and constraint $z \leq \alpha y + \gamma x$ is then satisfied for all $x \geq 0$. So the problem (8) becomes $\ell_{y=0} = \max\{-cx \mid 0 \leq x \leq a\} = \max\{0, ca\}$.

For $y = 1$, the problem (8) results in

$$\ell_{y=1} = \max wz - cx$$
$$z \leq \alpha + \gamma x,$$
$$0 \leq z \leq 1 \tag{9}$$
$$m \leq x \leq M,$$

By the definitions of α and γ we have $\alpha + \gamma x \leq 1$ for $m \leq x \leq M$. Hence we may relax condition $z \leq 1$ since it follows from $z \leq \alpha + \gamma x$.

Remaining constraints of the problem (9) form the polyhedron

$$P = \{x \mid z \leq \alpha + \gamma x, z \geq 0, m \leq x \leq M\},$$

having four vertices V_k, $k = 1, \ldots, 4$, with the following components $(x, z)_k$ and objective values $\ell_{y=1}^k$:

	V_1	V_2	V_3	V_4
$(x, z)_k$	$(m, 0)$	$(m, \alpha + \gamma m)$	$(M, 0)$	$(a, \alpha + \gamma a)$
$\ell_{y=1}^k$	$-cm$	$w(\alpha + \gamma m) - cm$	$-cM$	$w(\alpha + \gamma M) - cM$

Note that by definition of α, γ both $\alpha + \gamma m$ and $\alpha + \gamma M$ are positive, such that $\ell_{y=1}^2 > \ell_{y=1}^1$ and $\ell_{y=1}^4 > \ell_{y=1}^3$. Thus the optimal objective value of the problem (9) is

$$\ell_{y=1} = \max\{w(\alpha + \gamma m) - cm, \, w(\alpha + \gamma M) - cM\}.$$

Since $\ell = \max\{\ell_{y=0}, \ell_{y=1}\}$ we obtain for the optimal value of the problem (8):

$$\ell = \max\{0, \, w(\alpha + \gamma m) - cm, \, w(\alpha + \gamma M) - cM\}. \tag{10}$$

Consider now the Lagrangian dual problem to find the best (the smallest) Lagrangian bound

$$UB = \min_{u \geq 0} \sum_j \ell_j(u) + u^t b,$$

where $\ell_j(u)$ is defined similar to (8) for each project. This dual problem can be stated as a linear program in variables u, ℓ_j:

$$UB = \min \sum_j \ell_j + u^t b$$
$$\ell_j \geq w_j(\alpha_j + \gamma_j m_j) - c_j m_j,$$
$$\ell_j \geq w_j(\alpha_j + \gamma_j M_j) - c_j M_j,$$
$$c_j = (A^t u)_j,$$
$$u, \ell_j \geq 0,$$

with $F^* \leq UB$. Here $(A^t u)_j$ is the jth component of the vector $A^t u$ and variables ℓ_j are used for max in (10).

Suppose now that a number of sufficiently funded projects is limited from below. As it was shown in Proposition 1, $y_j = 1$ for $x_j \in [m, M]$ and $y_j = 0$ for $0 \leq x_j < m$. So we consider the problem (4)-(6) with the additional constraint

$$\sum_j y_j \geq p,$$

where p is the minimal number of sufficiently funded projects. Dualizing this constraint with multiplier $\theta \geq 0$, we get the corresponding Lagrangian problem

$$L(u, \theta) = u^t b - \theta p + \max\left\{\sum_j (w_j z_j - c_j x_j + \theta y_j)\right\}$$

subject to the constraints (5).

This problem also decomposes into J independent subproblems different from the problem (8) only in the objective: $\ell = \max wz - cx + \theta y$. It can also be analyzed by inspection, resulting in the following expression for its optimal objective:

$$\ell = \max\{0, \, w(\alpha + \gamma m) - cm + \theta, \, w(\alpha + \gamma M) - cM + \theta\}.$$

Respectively, the dual Lagrangian problem becomes

$$UB = \min \sum_j \ell_j + u^t b - \theta p$$

$$\ell_j \geq w_j(\alpha_j + \gamma_j m_j) - c_j m_j + \theta,$$

$$\ell_j \geq w_j(\alpha_j + \gamma_j M_j) - c_j M_j + \theta,$$

$$c_j = (A^t u)_j,$$

$$u, \ell_j, \theta \geq 0.$$

As was shown above, the number of sufficiently funded projects subject to constraints (5) can be presented by $\sum_j y_j$. Thus the bi-objective problem to maximize the number of funded projects and the portfolio quality can be stated as a bi-objective MILP:

$$\max \left\{ \sum_j w_j z_j, \sum_j y_j \right\}$$

subject to the constraints (5) and (6).

Due to the non-convexity of the feasible set to this problem, its efficient solution set in general can not be fully determined by parameterizing on $\pi \in [0,1]$ the weighted-sum problem

$$\max \pi \sum_j w_j z_j + (1-\pi) \sum_j y_j$$

subject to the constraints (5) and (6).

that is, there may exist efficient solutions that can not be reached even if the complete parameterization in π is attempted (see, e.g., [Alves and Clímaco, 2007] and the references therein). Meanwhile, the parameterization of this weighted-sum problem provides efficient points and can be useful to get an initial rough representation of the efficient set. Note, that the proposed Lagrangian techniques can also be applied to get easily computable bounds for the weighted-sum problem.

The characterization of all efficient points typically consists in introducing additional constraints into the weighted-sum problem. Generally, these constraints impose bounds on the objective function values, which can be regarded as a particularization of the general characterization provided by Soland (1979). The introduction of bounds on the objective function values enables the weighted-sum problem to compute all efficient solutions. Other characterizations based on reference points can be defined, using, for example, the augmented weighted Tchebycheff program (see, e.g., [Alves and Clímaco, 2007] and the references therein). We do not consider here the full characterization of efficient points for our bi-objective MILP leaving this interesting topic for our future research.

In the next section we present numerical results obtained by parameterizing on $\pi \in [0,1]$ the weighted-sum problem. For all problem instances the linear constraints $Ax \leq b$ represent the limit for the overall funding of the projects, as well as the bounds for the total funding in a particular area.

5 Numerical experiments

Five instances were considered with 40, 400, 1,200, 10,000, and 25,000 projects, while π was moved from zero to one with the step-size 0.01. The small instances were included for comparability and were constructed based on the previous works of Navarro (2001) and Fernández et al. (2004). The larger instances with 1,200, 10,000 and 25,000 projects, were generated with an instance-generation tool developed by Castro (2007). The ILOG CPLEX, version 9.0 was used as optimization tool. For all instances and values of π used, the run time of CPLEX was below 20

seconds on a four-processor SunFire server running the Solaris operating system, also version 9.0.

We compare our solutions with previously reported results only for the instances with 40 and 400 projects, since those methods cannot solve larger problems. For the 40-project instance, using $\pi = 1$ for comparability, we obtained a 28-project portfolio with quality indicator at 156.574, being very similar to that obtained by Navarro [2001]. Meanwhile, using $\pi = 0.43$ in our approach, the resulting portfolio quality was almost the same (154.704), but two more projects were supported as a 30-project portfolio was obtained.

For the case of 400 projects, we did not generate the instance identical to that used by Fernández et al. (2004) and thus not compared with their method in terms of portfolio quality. For larger instances we did not find references presenting a method capable to handle such a number of projects.

Optimizing the weighted-sum problem for a fixed π takes less than 20 seconds for all problem instances. Thus, for our bi-objective problem we could generate the Pareto front of 40 non-dominated solutions in a reasonable time using 40 different values of π.

In Figure 2, the Pareto front for the 10,000-project instance is shown. For other instances the shape of the curve behaved similarly. Such a shape justifies the use of the multi-objective model as a tool to search for a compromise between portfolio quality and the number of supported projects.
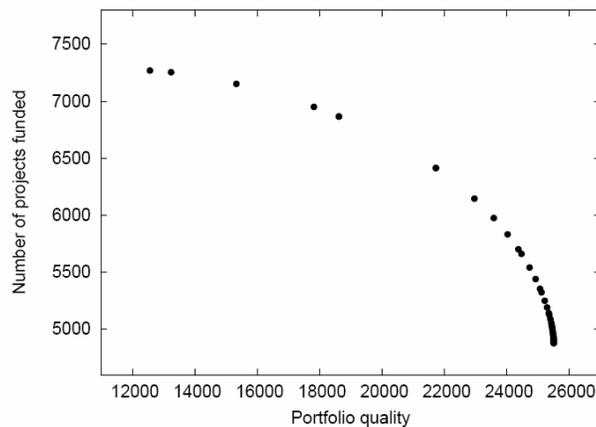

Fig. 2. The Pareto front of the 10,000-project instance

6 Conclusions

This paper presents a MILP model for the nonlinear multi-objective portfolio optimization for public R&D projects. The Lagrangian relaxation is studied to get simple computable bounds for the optimal objective. The weighted-sum scalarization of the bi-objective model is numerically tested for large and very large problem instances. In numerical tests only simple restrictions for funding were considered. Meanwhile, Lagrangian bounds derived in the paper are valid for the general form of funding constraints. Studying real problems with more complicated funding constraints is an interesting area for a future research.

The proposed model allows a decision maker to find a compromise between the quality and the size of the R&D projects portfolio. The experiments show that optimizing instances with up to 25,000 projects takes less than a minute, which superiors significantly the existing solutions techniques capable to handle in a reasonable time only up to 400 projects. The fast optimization is very important for an interactive decision-support system. This gives the decision maker an opportunity to explore different Pareto-optimal solutions and choose an acceptable compromise between the portfolio quality and the number of projects supported.

Acknowledgments

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